

# Decomposing Firm Value

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## Abstract

What are the economic determinants of the firm's market value? We answer this question through the lens of a generalized neoclassical model of investment with physical capital, quasi-fixed labor, and two types of intangible capital, knowledge capital and brand capital. We estimate the structural model using firm-level data on U.S. publicly traded firms and use the parameter values to infer the contribution of each input for explaining firm's market value in the last four decades. The model performs well in explaining both cross-sectional and time-series variation in firms' market values across all firms, with a time series  $R^2$  of 80% and a cross sectional  $R^2$  of 99%. We find that the relative importance of each input for firm value varies across industries. On average, physical capital accounts for 22.7% to 56.7% of firm's market value, installed labor force accounts for 18.2% to 40.1%, knowledge capital accounts for 0.9% to 33%, and brand capital accounts for 3.5% to 24%. These values also vary over time: the importance of physical capital for firm value has decreased in the last decades, while the importance of knowledge capital has increased, especially in high tech industries. Overall, our value decomposition provides direct empirical evidence supporting models with multiple capital inputs as main sources of firm value.

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# 1 Introduction

Understanding the economic determinants of a firm's market value is an important question that has attracted substantial research in finance and economics. We address this question through the lens of a generalized neoclassical model of investment with four different types of quasi-fixed inputs: physical capital (e.g., machines and plants), labor (a firm's installed labor force), and two types of intangible capital, namely knowledge capital (a firm's cumulated investment in innovation activities), and brand capital (a firm's cumulated investment in improving brand awareness). The rich model of the firm incorporates the evidence from Hall (2001), McGrattan and Prescott (2000), and Merz and Yashiv (2007) that, at the aggregate-level, intangible capital and installed labor force are important components of aggregate stock market values. Through structural estimation, and using data for a large cross section of publicly traded firms in the U.S. economy, we quantify the relative importance of the different capital inputs for understanding the level and the variation in firms' market values, both across industries and over time.

In the model, changing the quantity of the capital and labor inputs is costly, which we capture through standard adjustment cost functions. The adjustment costs for physical and intangible capital include planning and installation costs, and costs related with production being temporarily interrupted, among other costs. The adjustment costs for labor include the cost of hiring and firing workers, as well as the cost of training new workers. A firm's valuation ratio (market value of equity plus net debt divided by book value of capital inputs) is directly linked to the shadow price and quantity of each installed capital/labor input, and can be inferred from investment and hiring data through the specification of an adjustment cost function. The basic intuition for this result follows from standard neoclassical theory of investment (Hayashi 1982). At the optimum, firms invest in each capital input until the marginal cost of an additional unit equals the present value of its future benefits, that is, the shadow price of the input. We use this result to compute the market value of the installed capital and labor inputs as the product of the shadow price and the

corresponding stock variable. Under constant returns to scale, the market value of the firm is then the sum of the market value of all capital and labor inputs.

In the presence of capital and labor adjustment costs, the market value of the installed stocks of the physical capital and labor inputs is different from their book values, thus requiring estimates of adjustment costs (the book-values can be inferred from accounting data). In the case of labor, its book value is zero because firms do not sell nor buy workers as they do with capital goods, but its market value might be different from zero if it is costly to adjust the labor force. This is because, in equilibrium, firms extract rents from labor as a compensation for the costs of adjusting the labor force in the future. But if labor markets are competitive and frictionless (and there is no time-to-hire), labor inputs are paid their marginal product, and the rents from labor are zero. In this case, the contribution of the firms' installed labor force for the firm's market value is zero. The same logic applies to capital inputs, but here the book value of these inputs is not zero, even without adjustment costs, because firms can sell (or buy) capital. For example, in the one capital input model, without physical capital adjustment costs, the book value of the firm is equal to the book value of the physical capital stock.

Our estimation procedure is as follows. We estimate the structural parameters of the model by minimizing the distance between observed and model-implied valuation ratios (market value of equity and net debt-to-book value of the capital stocks) as in Belo, Xue, and Zhang (2013) (henceforth BXZ), who in turn follow the original estimation approach in Liu, Whited, and Zhang (2009) (henceforth LWZ). To abstract from idiosyncratic shocks responses that add noise to the firm level data, we estimate the model parameters using portfolio-level moments. We consider five portfolios sorted on firm's lagged valuation ratio (which is closely related to market-to-book ratio). We estimate the model across all firms in the economy, and also separately within different industries. Following Belo et al. (2017), we perform the estimation separately across low, medium-, and high labor-skill industries (henceforth low-, mid-, and high-skill industries). To a first approximation these industries correspond to low-, mid-, and high-tech sectors of the economy.

We modify the estimation procedure in BXZ and LWZ in two important ways. First, to estimate the model parameters, we target cross-sectional portfolio-level moments that do not require aggregating the data to construct portfolio-level aggregate valuation ratios. Specifically, for each portfolio, we target the cross-sectional median (computed across the firms in each portfolio) portfolio-level valuation ratio, which computation does not depend on a specific weighting scheme of the firms in each portfolio. This modification is important because, as we show using artificial data, the parameter estimates obtained using the BXZ/LWZ portfolio-level aggregation procedure are subject to an aggregation bias, and hence do not have a structural interpretation. We show that the procedure proposed here allow us to recover the firm-level structural parameters of interest, which is crucial to provide a proper decomposition of the market value of the firm.

Second, we estimate the model parameters by minimizing the sum of the squared difference (residuals) between the observed and the model-implied moments of the valuation ratios for each portfolio. Thus, our estimation procedure requires the model to match the realized time series of the observed valuation ratios as close as possible, and not just on average as in BXZ and LWZ. This is important in the context of our analysis because the contribution of each capital and labor input for firm value, as we document here, changes over time.

To take the model to the data, we need to measure the knowledge and brand capital stocks. Given their intangible nature, the data for these intangible capital inputs is not readily available from firms' balance sheet data. Following previous studies, we construct firm-level measures of knowledge capital stock and brand capital stock from firm-level accounting data on research and development (R&D), and data on advertising expenses, respectively. Accordingly, we interpret R&D expenditures as a firm's investment to generate new (or improve current) ideas. Similarly, we interpret advertising expenses as a firm's investment to enhance the value of brand names and brand awareness. We accumulate these expenditures using the perpetual inventory method to obtain the corresponding capital stocks.

Our main empirical findings can be summarized as follows. When the model is estimated across

all firms in the economy, the parameter estimates imply that, on average, physical capital accounts for 25.1% of firms' market value, installed labor force accounts for 49.1%, knowledge capital accounts for 21.9%, and brand capital accounts for 4.0%. Thus, on average, the non physical capital inputs account for the majority, about 75%, of firm's market value.

The estimated relative importance of the capital and labor inputs for firms' market values varies substantially across industries. On average, physical capital accounts for a large fraction of firm value in low-skill industries (about 57% of a firm's market value), but a significantly smaller fraction in high-skill industries (about 23% of a firm's market value). This result suggests that the standard one physical capital-input model is a more appropriate model for the firm in low-skill industries than for high-skill industries. Related, we show that the average fraction of firm value attributed to labor and knowledge capital increases with the average labor-skill level of the industries. In the low-skill industry, the fraction of firm value that can be attributed to labor and knowledge capital is on average only 18% and 1%, respectively, whereas in the high-skill industries these fractions are 41% and 33% respectively. This result suggests that adding labor and knowledge capital to the one capital input model is especially important for understanding the valuation of firms in high-skill industries. Finally, we find that the average fraction of firm value attributed to brand capital decreases with the average labor-skill level of the industry. Brand capital appears to be important in low-skill industries, where it accounts for about 24% of firm value, but not so much in high-skill industries where it accounts for only about 3.5% of firm value. Thus, our estimates show that, even though intangible capital is an important component of firm's market value across all industries, the type of intangible capital (knowledge or brand capital) that matters the most for firm value varies substantially across industries.

What explains the estimated firm value decomposition? We show that adjusting the four inputs in response to changing economic conditions is fairly costly. The parameter estimates imply that, consistent with Merz and Yashiv (2007), it is costly for a firm to adjust its labor force, especially in high-skill industries. Across all firms, we estimate that a firm's annual labor adjustment costs

represent on average about 22.3% of total annual sales. This figure is significantly higher than the average physical capital adjustment costs of about 3.8% of total annual sales. Similarly, our estimates show that it is costly to adjust both stocks of intangible capital. Brand capital adjustment costs are on average 0.5% of total annual sales, and knowledge capital adjustment costs are on average 9.1% of total annual sales.

The estimated size of the adjustment costs of the different capital and labor inputs varies substantially across industries. This fact helps understand why the relative importance of the capital and labor inputs for firms' value also varies across industries. The magnitude of the labor adjustment costs increases significantly with the average labor-skill level of the industry. The fraction of sales lost due to labor adjustment costs is on average 10% in the low-skill industries and 27% in high-skill industries. Thus, consistent with previous studies, we find that it is more costly to replace high-skill than low-skill workers (see discussion in related literature section below). Similarly, knowledge capital adjustment costs increase with the average labor-skill level of the industry. The fraction of sales lost due to knowledge capital adjustment costs is close to 0% in the low-skill industries, but about 21% in the high-skill industries. This positive relationship between adjustment costs and average labor-skill of the industry is reversed for physical capital and brand capital. The fraction of sales lost due to physical capital adjustment costs is on average 18.4% in the low-skill industries, and 11.2% in the high-skill industries. Similarly, the fraction of sales lost due to brand capital adjustment costs is on average 13.6% in the low-skill industries, and 1.6% in the high-skill industries.

In terms of model fit, the model performs well in explaining both the time-series and the cross-sectional variation of valuation ratios across the valuation ratio portfolios, with a time series  $R^2$  of 80% and a cross sectional  $R^2$  of 99%, when estimated across all firms in the economy. To help understand the good fit of the model and the relative importance of each capital and labor input for firm's valuation, we estimate restricted versions of the model using subsets of the capital and labor inputs. Consistent with BXZ, we find that the standard one-physical capital input

model does a reasonable job explaining the cross-sectional variation in the average valuation ratio across portfolios with a high cross-sectional  $R^2$  of 97%. Interestingly, the time-series  $R^2$  of the one capital-input model is also quite reasonable, 65%, suggesting that the estimation method used here is successful at dealing with measurement error problems that plague the investment-regressions that use firm-level data. Although reasonable, the time-series  $R^2$  of the one capital input model is 15 percentage points lower than that of the baseline model. Thus, we conclude that the benefit of incorporating additional quasi-fixed inputs in the neoclassical investment model comes primarily from improving the model's ability to capture the time-series variation in the valuation ratios. In addition, we show that the one capital-input model significantly overestimates the magnitude of physical-capital adjustment costs.

The comparison of the model fit across the different (restricted) specifications of the model further shows that adding labor and knowledge capital to the standard one-capital input model has a similar first order impact on the model fit. When quasi-fixed labor or knowledge capital is added to the one-capital input model, the time-series  $R^2$  increases from 65% to 76% in both cases. Across all firms, the improvement in the model fit due to the addition of brand capital is very modest. But across low-skill industries, the improvement is significant. In low skill industries, when brand capital is added to the one-capital input model, the time-series  $R^2$  increases from 53% to 61%.

We also investigate the time-series of the value, and value-shares, of each input. Across all firms, the importance of knowledge capital increased significantly over our sample period from 9.3% in the 1970s to 28.7% in the 2010s. The increased importance of knowledge capital crowded out the importance of the other inputs such as labor and, more significantly, physical capital. Over the sample period, the importance of labor inputs has decreased slightly from 55.2% in the 1970s to 47.9% in the 2010s. At the same time, the importance of physical capital for firm value has significantly decreased from 30.9% in the 1970s to 20.7% in the 2010s. The contribution of brand capital for firm value, when evaluated across all firms, is relatively small during the entire period, having decreased slightly in recent years. These trends are significantly more pronounced

in high-skill industries than in low-skill industries.

In addition, we investigate the risk properties of the capital and labor inputs by examining the correlation of the cyclical components of the value, and firm value shares, of each input with the cyclical component of aggregate sales, and also the volatility of each input value. We find that while the share of labor in firms' market value is procyclical, the shares of the other capital inputs is countercyclical, especially in high labor-skill industries. Thus, the importance of the labor input for firm's value is higher during good economic times. In addition, we find that, across all firms, the value of labor is the most volatile and most procyclical component of firm value. This finding suggests that the dynamics of labor inputs (and their associated labor market frictions) over time and across firms is an important determinant of firm's market value in financial markets.

Finally, we provide a series of robustness checks to establish the importance of non-physical capital inputs for firm value. We show that the main conclusions regarding the relative importance of the capital and labor inputs obtained using our proposed estimation procedure (which targets the time series of portfolio-level cross-sectional median) are similar to those obtained by targeting other sensible portfolio-level moments in the estimation, such as the cross-sectional equal-weighted mean, the inter-quartile range, and other moments of the cross sectional distribution of firm-level valuation ratios in each portfolio. In addition, we estimate a restricted version of the model with only physical capital, labor and brand capital (and excluding knowledge capital), using a different sample of firms that includes only firms that do not report R&D expenses, and find that, similar to the main model results, the non-physical capital inputs account for a large fraction (more than 50%) of firm's value in this alternative sample.

Taken together, our firm value decomposition provides direct empirical evidence supporting models with multiple capital inputs as main sources of firm value.

## *Related Literature*

Our work is related to the large literature on firm valuation.<sup>1</sup> Our approach is closely related to the supply approach to valuation developed in BXZ, but extended to a setup in which multiple and heterogeneous capital and labor inputs, not just physical capital, can contribute to a firm's market value. Importantly, our modified estimation method allows us to recover the firm-level structural parameters, which are crucial to perform valuation at the firm- and not just portfolio-level, thus substantially increasing the usefulness of the supply approach in practice. In a recent study, Gonçalves et al. (2017) also address the aggregation issues in the original LWZ portfolio-level aggregation approach (see also Zhang 2017 for a discussion of the aggregation bias in the standard tests of the investment-based model). Using a variation of one of the alternative estimation methods proposed here (the one that targets the portfolio-level cross-sectional mean), they show that the baseline investment-based model can simultaneously capture the variation in average returns across a large set of portfolios (value, momentum, profitability), and other empirical patterns in the cross section, with a stable set of parameter values, in contrast to the results obtained using the original portfolio-level aggregation procedure proposed in LWZ.

Our paper is related to the asset pricing literature on intangible capital and firm risk. Eisfeldt and Papanikolaou (2013) estimate the value of organization capital using a model of the sharing rule between a firm's owners and its key talent. They show that firms with more organization capital are riskier than firms with less organization capital. Following Lev and Radhakrishnan (2005), the authors measure organization capital using selling, general and administrative expenses (SG&A). Thus, their measure of organization capital is a broad concept: it includes the value of the labor force (because it includes the costs of training workers), knowledge capital (because it includes R&D expenses), brand capital (because it includes advertising expenses), among others. Since our goal is to decompose the value of the firm and understand the relative contribution of labor and the different intangible capital inputs for firms' market value, we do not use this broad

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<sup>1</sup>See BXZ for an overview of the firm valuation literature in Finance, Economics, and Accounting.

measure, and instead focus on measures of the separate components. This is important because, as we document here, the type of intangible capital (knowledge vs brand capital) that matters the most for firm value varies significantly across industries. Hansen, Heaton, and Li (2012) study the risk characteristics of intangible capital. Li and Liu (2012) and Vitorino (2014) study the importance of intangible capital in a q-theory model via structural estimation. We build on their work by considering a general model that includes both knowledge and brand capital, and, most importantly, frictions in the labor inputs. Hence, we provide a more accurate assessment of the contribution of each capital input to firm value, and investigate the business cycle properties of different capital inputs.

A growing literature has further shown the importance of intangible capital for corporate decisions. Falato, Kadyrzhanova, and Sim (2014), building on earlier work by Corrado, Hulten, and Sichel (2009) and Corrado and Hulten (2010), show that intangible capital is the most important firm-level determinant of corporate cash holdings, with the rise in intangible capital being a fundamental driver of the secular trend in U.S. corporate cash holdings over the last decades. We differ from these studies because our structural model allows us to measure the market value of the capital inputs, not just the book-values. As we show here, a firm value decomposition based on book-value of the capital inputs is significantly different from a decomposition based on the market value of the inputs.

Peters and Taylor (2017) propose a new measure of Tobin's Q that accounts for intangible capital, and show that their measure is a superior proxy for explaining total firm investment in physical and intangible capital. Our structural model of the firm, which also incorporates intangible capital, provides a quantitative decomposition of Tobin's Q into the value of each capital input according to the optimal corporate policies including labor hiring, and investment in physical and intangible capital. In addition, Peters and Taylor (2017) document that the investment-q relation works best in high tech sectors. Andrei, Mann, and Moyen (2018) confirm this finding and show that it can be rationalized in an augmented investment model with corporate learning about firms'

cash flows. Consistent with these findings, we show that an augmented investment model with two types of intangible capital and quasi-fixed labor inputs matches the data in the high tech sector particularly well, further improving the fit of the one capital-input model.

The findings in our paper are also related to the large literature that tries to understand the trend in the labor share in the economy. The change in the importance of labor for firm value that we document here resembles the evidence in Hartman-Glaser, Lustig, and Xiaolan (2017) who show that the cross-sectional average labor share of publicly traded firms has increased over time in the U.S. economy (in contrast with the well documented decrease of the aggregate labor share over the same sample period, as noted in Elsby, Hobijn, and Şahin 2013, Karabarbounis and Neiman 2013, among others). The difference is that we compute the importance of the value of labor for firm value, and not for value added as in Hartman-Glaser, Lustig, and Xiaolan (2017).

An important strand of the asset pricing literature documents the effect of labor market frictions on stock returns and firm value.<sup>2</sup> The theoretical approach in this paper is related to the work of Merz and Yashiv (2007), who build upon the earlier work by Cochrane (1991). Merz and Yashiv (2007) consider an aggregate representative firm facing adjustment costs in both capital and labor, and focus on the estimation of the production and adjustment cost functions. They show that adding labor adjustment costs substantially improves the model’s ability to capture the dynamics of the aggregate stock market value. We build on the Merz and Yashiv (2007)’s setup by including two additional types of costly intangible capital. Extending the model to the firm-level further allows us to exploit not only time-series data, but also firm-level cross-sectional data. Belo, Lin, and Bazdresch (2014) add labor adjustment costs to Zhang (2005)’s model and show that labor hiring negatively predicts future returns in the cross section both in model simulations and in the data. In our work, we focus on equity valuation ratios and we provide a structural estimation of the frictions in (physical and intangible) capital and labor markets.

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<sup>2</sup>A partial list of studies linking labor market variables to asset prices include Mayers (1972), Fama and Schwert (1977), Campbell (1996), Jagannathan and Wang (1996), Jagannathan, Kubota, and Takehara (1998), Santos and Veronesi (2005), Boyd, Hu, and Jagannathan (2005), and Lustig and Van Nieuwerburgh (2008). The interpretation of the empirical facts in these studies is silent about the production-side of the economy (technology).

Our work is also related to the large literature on labor demand and capital investment which investigates the importance of capital and labor adjustment costs to explain investment and hiring dynamics.<sup>3</sup> The estimated economic magnitude of adjustment costs is still subject to debate. For example, Shapiro (1986) shows that large estimates of labor adjustment costs are important to match investment and hiring dynamics, particularly for non production workers. Hall (2004), however, estimates both capital and labor adjustment costs to be negligible at the two-digit SIC industry level. We add to this literature by providing structural estimates of adjustment costs for multiples types of capital and labor inputs based on financial market data.

Finally, our paper also contributes to the literature on the importance of capital heterogeneity. Abel (1985) provides closed-form solutions for firm market value in a q-theory model with several factors of production, and Abel and Eberly (2001) provide empirical evidence on the relevance of capital heterogeneity. Using a dataset of Japanese firms, Hayashi and Inoue (1991) find strong empirical support for the relationship between aggregate capital growth and Tobin’s Q derived in a model with multiple capital goods. Similarly, Chirinko (1993) estimates an investment model with multiple capital inputs and adjustment technologies, and find significant evidence in favor of capital heterogeneity. Our firm value decomposition provides additional direct empirical evidence supporting models with multiple capital inputs.

## 2 The Model of the Firm

This section solves the optimal investment decision of a firm. The model is a neoclassical model of the firm as in LWZ/BXZ (we use their notation whenever possible), extended to a setup with several quasi-fixed inputs. Time is discrete and the horizon infinite. Firms choose costlessly adjustable inputs each period, while taking their prices as given, to maximize operating profits (revenues minus

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<sup>3</sup>See, for example, on capital: Cooper and Haltiwanger (1997), Caballero et al. (1995), Cooper, Haltiwanger, and Power (1999) and Cooper and Haltiwanger (2006); on labor: Hamermesh (1989), Bentolila and Bertola (1990), Davis and Haltiwanger (1992), Caballero and Engel (1993), Caballero, Engel, and Haltiwanger (1997), Cooper, Haltiwanger, and Willis (2015); on joint estimation of capital and labor adjustment costs Shapiro (1986), Galeotti and Schiantarelli (1991), Hall (2004), Merz and Yashiv (2007) and Bloom (2009). Bond and Van Reenen (2007) survey the literature and Hamermesh (1996) reviews a set of direct estimates of labor adjustment costs.

the expenditures on these inputs). Taking these operating profits as given, firms optimally choose the physical and intangible capital investments, hiring, and debt to maximize the market value of equity. To save notation, we denote a firm's  $i$  set of capital (and labor) inputs as  $K_{i,t}$ . This set includes the physical capital stock ( $K_{i,t}^P$ ), labor stock ( $L_{i,t}$ ), knowledge capital stock (an intangible, and hence unmeasured (U) capital input in firm's accounts,  $U_{i,t}^K$ ), and brand capital stock (another intangible capital input,  $U_{i,t}^B$ ). Similarly, we denote a firm's  $i$  set of investment in the capital inputs as  $I_{i,t}$ . This set includes the investment in physical capital ( $I_{i,t}^P$ ), investment in labor stock, that is, gross hiring ( $H_{i,t}$ ), investment in knowledge capital ( $I_{i,t}^K$ ), and investment in brand capital  $I_{i,t}^B$ .

## 2.1 Technology

The operating profits function for firm  $i$  at time  $t$  is  $\Pi_{it} \equiv \Pi(K_{it}, X_{it})$ , in which  $X_{it}$  denotes a vector of exogenous aggregate and firm-specific shocks. We assume that the firm has a production function with constant returns to scale.

The law of motion of the firm's capital inputs and labor force are given by:

$$K_{it+1}^P = I_{it}^P + (1 - \delta_{it}^P)K_{it}^P \quad (1)$$

$$L_{it+1} = H_{it} + (1 - \delta_{it}^L)L_{it} \quad (2)$$

$$U_{it+1}^K = I_{it}^K + (1 - \delta_{it}^K)U_{it}^K \quad (3)$$

$$U_{it+1}^B = I_{it}^B + (1 - \delta_{it}^B)U_{it}^B, \quad (4)$$

where  $\delta_{it}^P$ ,  $\delta_{it}^K$  and  $\delta_{it}^B$  are the exogenous depreciation rates of physical, knowledge and brand capital, respectively.  $\delta_{it}^L$  is the employee quit rate, i.e. the rate at which the workers leave the firm for voluntary reasons.

Firms incur adjustment costs when investing. The augmented adjustment costs function, denoted  $C_{it} \equiv C(I_{i,t}, K_{i,t})$ , is increasing and convex in investment/hiring, decreasing in the capital stocks, and has constant returns to scale.

## 2.2 Taxable Profits and Firms' Payouts

We allow firms to finance investments with debt. At the beginning of time  $t$ , firm  $i$  issues an amount of debt, denoted  $B_{it+1}$ , which must be repaid at the beginning of time  $t + 1$ . Let  $r_{it}^B$  denote the gross corporate bond return on  $B_{it}$ . We can write taxable corporate profits as operating profits minus depreciation, adjustment costs, and interest expense:

$$\Pi_{it} - I_{it}^K - I_{it}^B - W_{it}L_{it} - \delta_{it}^K K_{it} - C_{it} - (r_{it}^B - 1)B_{it}.$$

Let  $\tau_{it}$  be the corporate tax rate. We define the payout of firm  $i$  as:

$$D_{it} \equiv (1 - \tau_t)[\Pi_{it} - C_{it} - I_{it}^K - I_{it}^B - W_{it}L_{it}] - I_{it}^P + B_{it+1} - r_{it}^B B_{it} + \tau_t \delta_{it}^K K_{it}^P + \tau_t (r_{it}^B - 1)B_{it}, \quad (5)$$

in which  $\tau_t \delta_{it}^K K_{it}^P$  is the depreciation tax shield and  $\tau_t (r_{it}^B - 1)B_{it}$  is the interest tax shield. Adjustment costs are expensed, consistent with treating them as foregone operating profits.

## 2.3 Equity Value

Firm  $i$  takes the stochastic discount factor, denoted  $M_{t+1}$ , from period  $t$  to  $t + 1$  as given when maximizing its cum-dividend market value of equity:

$$V_{it} \equiv \max_{\{\mathbf{I}_{it+\Delta t}, \mathbf{K}_{it+\Delta t+1}, \mathbf{B}_{it+\Delta t+1}\}_{\Delta t=0}^{\infty}} E_t \left[ \sum_{\Delta t=0}^{\infty} M_{t+\Delta t} D_{it+\Delta t} \right], \quad (6)$$

subject to a transversality condition given by  $\lim_{T \rightarrow \infty} E_t[M_{t+T} B_{it+T+1}] = 0$ .

Let  $P_{it} \equiv V_{it} - D_{it}$  be the ex-dividend equity value. Appendix A shows that firms' value maximization implies that:

$$P_{it} + B_{it+1} = q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K K_{it+1}^K + q_{it}^B K_{it+1}^B, \quad (7)$$

in which

$$q_{it}^P \equiv 1 + (1 - \tau_t) \partial C_{it} / \partial I_{it}^P \quad (8)$$

$$q_{it}^L \equiv (1 - \tau_t) \partial C_{it} / \partial H_{it} \quad (9)$$

$$q_{it}^K \equiv (1 - \tau_t) [1 + \partial C_{it} / \partial I_{it}^K] \quad (10)$$

$$q_{it}^B \equiv (1 - \tau_t) [1 + \partial C_{it} / \partial I_{it}^B.] \quad (11)$$

The variables  $q_{it}^P, q_{it}^L, q_{it}^K$  and  $q_{it}^B$  measure the shadow prices of physical capital, labor force, knowledge capital, and brand capital, respectively.

Equation (7) provides a formula to decompose the firm value as the sum of the value of the firm's installed labor and capital inputs. Specifically, the fraction of firm value that is attributed to these inputs is as follows:

$$\mu_{it}^P = \frac{q_{it}^P K_{it+1}^P}{q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K U_{it+1}^K + q_{it}^B U_{it+1}^B} \quad (12)$$

$$\mu_{it}^L = \frac{q_{it}^L L_{it+1}}{q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K U_{it+1}^K + q_{it}^B U_{it+1}^B} \quad (13)$$

$$\mu_{it}^K = \frac{q_{it}^K K_{it+1}^K}{q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K U_{it+1}^K + q_{it}^B U_{it+1}^B} \quad (14)$$

$$\mu_{it}^B = \frac{q_{it}^B K_{it+1}^B}{q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K U_{it+1}^K + q_{it}^B U_{it+1}^B}. \quad (15)$$

A fundamental goal of the empirical analysis is to characterize these weights including their variation over time and across industries.

### 3 Econometric Methodology

In this section we specify the functional forms and the estimation method used to obtain the structural parameters. In addition, we describe the data and report descriptive statistics for the key variables in the model.

### 3.1 Functional Forms

We consider the following flexible functional form for the adjustment costs function:

$$C_{it} = \frac{1}{\nu_P} \left| \theta_P \frac{I_{it}^P}{K_{it}^P} \right|^{\nu_P} K_{it}^P + \frac{1}{\nu_L} \left| \theta_L \frac{H_{it}}{L_{it}} \right|^{\nu_L} W_{it} L_{it} + \frac{1}{\nu_K} \left| \theta_K \frac{I_{it}^K}{U_{it}^K} \right|^{\nu_K} U_{it}^K + \frac{1}{\nu_B} \left| \theta_B \frac{I_{it}^B}{U_{it}^B} \right|^{\nu_B} U_{it}^B, \quad (16)$$

in which  $W_{it}$  is the wage rate (which the firm takes as given),  $\theta_P, \theta_L, \theta_K, \theta_B > 0$  are the slope adjustment cost parameters, and  $\nu_P, \nu_L, \nu_K, \nu_B > 1$  are the curvature adjustment cost parameters. Labor adjustment costs are proportional to the firm's wage bill, as in Bloom (2009). This helps to make the units of labor adjustment costs (measured in number of workers) similar to the other capital inputs which are measured in (real) dollar values, an adjustment that is important for the empirical results below. This specification nests the standard quadratic functional forms as special cases when the curvature parameters are equal to two.<sup>4</sup>

The absolute value specification of the adjustment cost function allows for negative investment rates and improves the stability of the estimation of curvature parameters.<sup>5</sup> This functional form generalizes the one-capital input functional form specification used in BXZ to multiple inputs.<sup>6</sup>

The adjustment cost function in equation (16) implies that the shadow prices of the capital inputs are given by:

$$q_{it}^P \equiv 1 + (1 - \tau_t) \theta_P^{\nu_P} \left| \frac{I_{it}^P}{K_{it}^P} \right|^{\nu_P - 1} \text{sign} \left( \frac{I_{it}^P}{K_{it}^P} \right) \quad (17)$$

$$q_{it}^L \equiv (1 - \tau_t) \theta_L^{\nu_L} \left| \frac{H_{it}}{L_{it}} \right|^{\nu_L - 1} \text{sign} \left( \frac{H_{it}}{L_{it}} \right) W_{it} \quad (18)$$

$$q_{it}^K \equiv (1 - \tau_t) \left[ 1 + \theta_U^{\nu_K} \left| \frac{I_{it}^K}{U_{it}^K} \right|^{\nu_K - 1} \text{sign} \left( \frac{I_{it}^K}{U_{it}^K} \right) \right] \quad (19)$$

$$q_{it}^B \equiv (1 - \tau_t) \left[ 1 + \theta_U^{\nu_B} \left| \frac{I_{it}^B}{U_{it}^B} \right|^{\nu_B - 1} \text{sign} \left( \frac{I_{it}^B}{U_{it}^B} \right) \right]. \quad (20)$$

<sup>4</sup>We place the slope adjustment cost parameters inside the absolute values of equation (16) to make the units of the slope adjustment cost parameters independent of the curvature parameter. This improves identification and stability during the estimation. See Belo, Xue, and Zhang (2013) for a similar approach.

<sup>5</sup>When the curvature parameters are greater than one,  $\nu_i > 1$ , this function is continuous along its entire domain including at zero since left and right derivatives at zero coincide. See also Kogan and Papanikolaou (2012) for a similar specification in the context of a one capital input model.

<sup>6</sup>Although not explicitly stated, BXZ also use absolute value specification to deal with negative investment rates observed in the data.

We use the sign function to express the equilibrium shadow prices of each of the capital inputs in a compact manner, that is, using one equation, instead of a piecewise function. This is because, given the absolute value specification, the signs associated with the investment and hiring rate terms switch depending on whether the input-specific investment or hiring rate is positive or negative.

### 3.2 Estimation Procedure

Equation (7) links firm value to the value of its labor and capital inputs. Since firm values are not necessarily stationary, it is useful to scale this variable for estimation purposes. We divide both sides of equation (7) by the sum of a firm’s capital inputs (not including labor), which we denote as  $A_{it+1}$ , a measure of the firm’s total (effective) assets given by  $A_{it+1} \equiv K_{it+1}^P + U_{it+1}^K + U_{it+1}^B$ . We do not include labor inputs to compute total assets because labor is measured in different units (number of workers as opposed to dollars in real terms). Hence, we write a firm’s valuation ratio ( $VR_{it} \equiv (P_{it} + B_{it+1}) / A_{it+1}$ ) as:

$$VR_{it} = q_{it}^P \frac{K_{it+1}^P}{A_{it+1}} + q_{it}^L \frac{L_{it+1}}{A_{it+1}} + q_{it}^K \frac{K_{it+1}^K}{A_{it+1}} + q_{it}^B \frac{K_{it+1}^B}{A_{it+1}}. \quad (21)$$

The left-hand side (LHS) of equation (21) can be directly measured in the data from equity price data and debt data (and measures of the capital stocks, which we discuss below). The right hand side (RHS) of the equation (21) is the predicted valuation ratio from the model,  $\widehat{VR}_{it}$ , which depends on the model parameters.

#### Aggregation Issues

Equation (21) establishes an exact relationship between a firm’s observed valuation ratio and the model-implied valuation ratio. We perform the estimation at the portfolio-level as in BXZ, which in turn follow the original approach in LWZ. The use of portfolio-level data has several appealing features. First, the focus on portfolio-level moments allows us to reduce the noise in the firm-level data. In addition, the portfolio-level moments are less sensitive to firm entry and exit, and are less likely to be affected by missing observations at the firm-level. This is an important consideration

in the context of our application because the R&D and advertising expenses data necessary to construct the knowledge capital and brand capital stocks are missing for a nontrivial fraction of the firms in Compustat (as discussed in Section 3.3 below). Unlike LWZ/BXZ, however, we estimate the model parameters by targeting cross-sectional portfolio-level moments that do not require aggregating the data to construct a portfolio-level aggregate valuation ratio, thus allowing us to recover the firm-level structural parameters.

**Aggregation in LWZ/BXZ** Before explaining our estimation method, it is useful to revisit the aggregation procedure in LWZ/BXZ because it is the limitation of their procedure in the context of our research question that justifies the use of an alternative method.<sup>7</sup> Following the approach in LWZ/BXZ, one would estimate the valuation equation at the portfolio-level by first computing the portfolio-level characteristics (e.g., the portfolio-level investment rates), and then plugging these characteristics directly in the valuation equation (21) to obtain the observed and the model-implied valuation ratios. Specifically, in year  $t$ , the portfolio  $j$  investment rate in physical capital is computed as:

$$\frac{I_{jt}^K}{K_{jt}} = \frac{\sum_i I_{j,i,t}^K}{\sum_i K_{j,i,t}}, i \in \text{Portfolio } j \quad (22)$$

which is then substituted in equation (17) to obtain the portfolio-level shadow price of the physical capital stock. Similarly, the portfolio level observed valuation ratio and capital stocks are given by:

$$VR_{jt} = \frac{\sum_i (P_{it} + B_{it+1})}{\sum_i A_{it}}$$

$$K_{jt} = \sum_i K_{j,i,t}, i \in \text{Portfolio } j.$$

The estimation would then proceed to estimate the parameter values by the Generalized Method of Moments (GMM) under the identification assumption that the model errors, computed as the difference between the portfolio-level aggregated observed and model-implied valuation ratios, are

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<sup>7</sup>Liu, Whited, and Zhang (2009) estimate the model predicted investment returns rather than valuation ratios using portfolio-level aggregated data. The two are closely related, however, because, to a first order approximation, the investment return is the valuation equation in first differences.

on average zero.

The LWZ/BXZ approach provides a powerful framework for identifying robust links between valuation ratios/stock returns and portfolio-level characteristics. In addition, this approach averages out measurement error in firm-level data in a convenient and elegant manner. Unfortunately, the aggregation procedure in the LWZ/BXZ approach complicates the interpretation of the parameter estimates. Specifically, by using the portfolio-level characteristics computed as in equation (22) to construct the shadow price of the capital input in equations (17), the procedure does not guarantee the recovery of the true firm-level structural parameters because the shadow prices of the capital inputs are, in general, nonlinear functions of the firm characteristics. Appendix B provides a more detailed analysis of this issue and provides estimates of the aggregation bias for a particular calibration of the adjustment costs function in the context of a one capital input model.<sup>8</sup>

**Our Alternative Estimation Procedure** To recover the firm-level structural parameters we thus modify the econometric approach proposed in LWZ. As noted, in theory, any moment of the observed firm-level valuation ratios in equation (21) should be equal to any corresponding moments of the model-implied firm-level valuation ratios. Thus, we target cross-sectional portfolio-level moments that do not require aggregating the data to construct a portfolio-level aggregate valuation ratio, hence avoiding the aggregation bias. Specifically, in each year, we compute the portfolio-level valuation ratio by taking the cross-sectional median of the firm-level observed and model-implied valuation ratios, which we refer to as cross-sectional median (XSMED) estimation. Since the median is insensitive to outliers, it is a natural moment to use in the estimation to mitigate the influence of large outliers in firm-level data. The median is also better suited to describe the economic behavior of the typical firm in the economy, and hence provide a better link to the model of the firm used here.<sup>9</sup> We use the XSMED estimation procedure to produce the baseline results.

We perform the estimation of the valuation equation (21) under the standard assumption

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<sup>8</sup>Belo and Deng (2018) provide a general analysis of the aggregation bias and other economic issues in the context of empirical tests of investment-based models.

<sup>9</sup>Our relatively simple model is less appropriate for the valuation of superstar firms like Apple or Facebook.

that the portfolio-level valuation ratio moments (which, in the baseline specification, is the cross sectional median valuation ratio across all firms in each portfolio) are observed with error by the econometrician:

$$VR_{jt}^{MOM} = \widehat{VR}_{jt}^{MOM}(\Theta) + \varepsilon_{jt}, \quad (23)$$

where  $\widehat{VR}_{jt}^{MOM}(\Theta)$  denotes the model-implied portfolio-level moment (MOM) of the cross-section of firm-level valuation ratios for the firms in portfolio  $j$  at time  $t$ ,  $\Theta$  represents the vector of structural parameters including an intercept, i.e.  $\Theta = [\theta_P, \theta_L, \theta_K, \theta_B, \nu_P, \nu_L, \nu_K, \nu_B, \alpha]$ , and  $\varepsilon$  captures measurement error in the portfolio-level moments.<sup>10</sup> The parameter  $\alpha$  is an intercept that we include in the estimation to allow for nonzero average measurement error. Based on equation (23), we estimate the model parameters by nonlinear least squares (NLLS), that is, we minimize the distance between the portfolio-level observed and model-implied valuation ratios moments:

$$\widehat{\Theta} = \arg \min_{\Theta} \frac{1}{TN} \sum_{t=1}^T \sum_{j=1}^N \left( VR_{jt}^{MOM} - \widehat{VR}_{jt}^{MOM}(\Theta) \right)^2.$$

Thus, unlike LWZ and BXZ, who estimate the model parameters by matching the time series means of the observed and model-implied portfolio valuation ratios, the use of NLLS in our estimation requires the model to match the realized time series of the observed cross sectional moments of the valuation ratios as close as possible. We then compute bootstrapped standard errors that are robust to cross-sectional and time-series correlation using 20% of the sample with replacement. As shown by Cameron and Miller (2010) bootstrapping controls for the fact that errors can be correlated across portfolios and within portfolios over time.

As a robustness check, we also consider an alternative estimation approach which targets other sensible portfolio-level moments in the estimation, such as the cross-sectional equal-weighted mean (XSEW). In Appendix B we show that, under the assumptions described here, the baseline approach (XSMD) and the robustness (XSEW) recover the underlying firm-level structural parameters. In

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<sup>10</sup>Mismeasured components of the valuation ratio such as the market value of debt and the capital inputs can be better observed by firms than by econometricians. Furthermore, the intrinsic value of equity can temporarily diverge from the market value of equity.

addition, as an additional robustness check reported below, we further investigate if the parameter estimates differ significantly from the baseline estimation if we target other portfolio-level moments, such as the inter-quartile range, and the 25<sup>th</sup> and 75<sup>th</sup> percentiles (not just the 50<sup>th</sup> percentile as in the baseline approach) of the cross sectional distribution of the firm-level valuation ratios in each portfolio.

### 3.3 Data and Test Assets

*Sample selection:* Our sample consists of all common stocks on NYSE, Amex, and Nasdaq from 1950 to 2016 (our estimation period starts in 1975 but we use data prior to 1975 to construct the initial intangible capital stocks as described below). The firm-level data are from the Center for Research in Security Prices (CRSP)/Compustat Merged (CCM) – Fundamentals Annual database. We limit our analysis to firms incorporated in the US (Compustat fic=“USA”) that trade on major stock exchanges (NYSE, AMEX, and NASDAQ) (CRSP exchange codes 1, 2, and 3), for which the native currency is US dollars (Compustat curcd=“USD”), and that have information on their ordinary common shares traded (CRSP share codes 10 and 11). We exclude firms with primary standard industrial classifications (SIC) between 4900 and 4999 (regulated utilities) and between 6000 and 6999 (financial services).

*Physical capital data:* The initial physical capital stock,  $K_{i0}^P$ , is given by net property, plant, and equipment (Compustat data item PPENT). The capital depreciation rate,  $\delta_{it}^K$ , is the amount of depreciation (item DP) divided by the beginning of the period capital stock. We measure the capital stock at current prices. Specifically, we construct an investment-price adjusted capital stock that accounts for changes in the real cost of physical capital investment by repricing last period’s capital stock using today’s price of investment:  $K_{t+1}^P = K_t^P(1 - \delta_t) \frac{P_{t+1}}{P_t} + I_{t+1}$ . Following Zhang (2017) we infer physical capital investment from the the law of motion of capital by inverting the law of motion equation and solving for investment. This procedure guarantees that the investment and

physical capital data are consistent with the law of motion for physical capital in the model.<sup>11</sup>

We construct the price index for physical capital as follows. The Bureau of Economic Analysis (BEA) provides a price index for “Gross Private Domestic Investment: Fixed Investment: Nonresidential” (series id A008RD3Q086SBEA in FRED) which includes structures, equipment and intellectual property products (and also provides separate indices for each of these three items) but we need an index for physical capital that includes structures and equipment and excludes intellectual property products because these correspond to intangible capital. We calculate the implicit price deflator for physical capital in the same manner as the BEA constructs implicit price deflators by dividing the current-dollar value of a series by its calculated real value. Specifically, we first recover the real value for structures and equipment by dividing the nominal values for these series reported by the BEA in the NIPA table 5.3.5 by the price indices reported by the BEA in the NIPA table 5.3.4. We then construct an aggregate price index for physical capital that includes both structures and equipment by dividing the sum of the nominal investment in structures and equipment by the sum of the real investment in structures and equipment.

*Labor data:* The labor stock,  $L_{it}$ , is number of employees (item EMP in Compustat). The labor market data on wage rate and labor quit rate is not available at the firm level (the firm level wage bill data in Compustat is missing for more than 80% of the firms in our sample). Thus, we measure these variables at the industry level as follows:

*Wage rate per worker:* We measure  $W_{it}$  using annual data from the BEA, National Income and Product Accounts (NIPA), Section 6. We compute the industry level (annual) wage rate per worker as the ratio of the total compensation of employees (which includes wage and salary accruals and supplements to wages and salaries) to the total number of employees in the industry. We use compensation of employees by industry from Tables 6.2B-D and full-time and part-time employees

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<sup>11</sup>Many studies measure investment in physical capital,  $I_{it}^P$ , as capital expenditures (item CAPX) minus sales of property, plant, and equipment (item SPPE), and set SPPE to zero if missing. As shown in Zhang (2017), this procedure generates investment series that violate the assumed law of motion of physical capital in several observations. The main reason for this fact is that CAPX excludes acquisitions, that is, increases in the firm’s capital stock due to the acquisition of other firms.

by industry from Tables 6.4B-D. Based on the industry description in the BEA tables we created a mapping between the wage data and the SIC 1987 and NAICS 2002 codes which we then used to merge the wage data with the firm-level data from Compustat/CRSP.

*Employee quit rate:* We measure annual employee quit rate  $\delta_{it}^L$  using data for 16 major industry groups based on NAICS codes from the Job Openings and Labor Turnover Survey (JOLTS) available from the Bureau of Labor Statistics (BLS). Because this data is only available since 2001, we extend the data backwards as follows. We estimate a time-varying quit rate by regressing, for each major industry group in JOLTS, the industry level quit rates on real GDP growth, unemployment rate, the labor vacancy rate, and a measure of labor market tightness.<sup>12</sup> The fit of the regression for each industry is quite good, with a time series  $R^2$  above 85% in most industries. For each industry, we then extend the quit rate back to cover the entire sample prior to 2001. We also use the same procedure to estimate a time-varying aggregate JOLTS quit rate for the industry group TOTAL PRIVATE (i.e. overall), and assign this rate to firms that belong to industries not covered in JOLTS or with missing industry code. This procedure allows us to have both cross-sectional and time-varying variation in the employee quit rate.

*Knowledge capital data:* Following Falato, Kadyrzhanova, and Sim (2014) we construct the firm's stock of knowledge capital from past expenditures data on research and development (R&D) (item XRD in Compustat) and using the perpetual inventory method as follows:<sup>13</sup>

$$U_{t+1}^K = U_t^K (1 - \delta^K) \frac{P_{t+1}^K}{P_t^K} + I_{t+1}^K, \quad (24)$$

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<sup>12</sup>For the real GDP growth we use the series: Real Gross Domestic Product (U.S. Bureau of Economic Analysis, series A191RL1A225NBEA, retrieved from FRED, Federal Reserve Bank of St. Louis; <https://fred.stlouisfed.org/series/A191RL1A225NBEA>, April 30, 2018). As for the unemployment rate we use the series: Civilian Unemployment Rate [UNRATE](U.S. Bureau of Labor Statistics, retrieved from FRED, Federal Reserve Bank of St. Louis; <https://fred.stlouisfed.org/series/UNRATE>, April 30, 2018.). For the labor vacancy rate we use the Help Wanted Index (HWI) referenced in Barnichon (2010) and provided in Regis Barnichon's website. The HWI is provided at the monthly level, in the regressions we use the yearly average. We construct a measure of labor market tightness as the ratio of the vacancy rate and the unemployment rate.

<sup>13</sup>See also Lev and Radhakrishnan (2005), Eisefeldt and Papanikolaou (2013), and Li and Liu (2012) for similar applications. The Bureau of Economic Analysis uses a similar methodology to construct a stock of Research and Development capital, see Sliker (2007).

where  $P_t^K$  is the BEA price index for intellectual property products, R&D, from the Federal Reserve Economic Data (FRED) database.<sup>14</sup> To implement the law of motion in equation (24) we must choose an initial stock and a depreciation rate. Using the perpetual inventory method, we choose the initial stock according to:

$$U_0^K = \frac{I_0^K}{g^K + \delta^K - \pi^K(1 - \delta^K)},$$

in which  $I_0^K$  is the firm's investment in knowledge capital in the first year in the sample, and  $\pi^K$  is the average (net) growth rate of the price index for R&D, which is 3.2% in the sample period used for the estimation. We choose  $g^K$  to be industry specific (we consider three labor skill industries), and equal to the average growth rate of the R&D investments in that industry. Accordingly, we set  $g^K = 16.8\%$  for firms in the low-skill industry,  $g^K = 15.9\%$  for firms in the mid-skill industry, and  $g^K = 24.7\%$  for firms in the high-skill industry. For knowledge capital depreciation rate, we use the recommended depreciation rates of R&D assets based on the BEA-NSF dataset as calculated by Li (2012) and reported for each industry on Table 4, column 3. For the companies/industries not reported in Li (2012) we use a 15% depreciation following Peters and Taylor (2017). Once we have the initial capital stock, we iterate forward using the appropriate depreciation rate, R&D expenses, and investment price index. The investment rate on knowledge capital is then given by the ratio of the current period investment and the beginning of the period corresponding knowledge capital stock  $I_t^K/K_t^K$ .

We treat the missing R&D expenditure data as follows. For firms that never report R&D data, we consider these firms to be non-R&D firms and hence we exclude these firms from the main sample because the model with knowledge capital does not apply to these firms (we consider these firms later in a robustness check where we estimate, separately, for these firms, a model without knowledge capital). For firms in which R&D expenditure data is missing for some, but not all,

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<sup>14</sup>Specifically, we use the annual series "Gross Private Domestic Investment: Fixed Investment: Nonresidential: Intellectual Property Products: Research and Development (chain-type price index), Index 2009=100" (Y006RG3A086NBEA) provided by the BEA.

years and sales and general administrative expenses (SG&A) data is not missing, we impute the R&D expenditure data for the missing years based on the firm-level average R&D expenses to SG&A ratio. The rationale for this procedure is the fact that SG&A data includes R&D expenses (together with other expenses). In robustness checks, we find that our main findings are robust to excluding firm-year observations with imputed R&D data, but using this imputation method allows us to expand the sample size.

*Brand capital data:* The construction of the brand capital stock is analogous to the construction of the knowledge capital stock. Following Belo, Lin, and Vitorino (2014) we construct the firm’s stock of the brand capital from past expenditures data on advertising expenses (item XAD) and using the perpetual inventory model as follows:

$$U_{t+1}^B = U_t^B(1 - \delta^B) \frac{P_{t+1}^B}{P_t^B} + I_{t+1}^B, \quad (25)$$

where  $P_t^B$  is the advertising industry’s output price index (PPI), available from the Bureau of Labor Statistics.<sup>15</sup> Using the perpetual inventory method, we choose the initial stock according to:

$$U_0^B = \frac{I_0^B}{g^B + \delta^B - \pi^B(1 - \delta^B)},$$

in which  $I_0^B$  is the firm’s investment in brand capital in the first year in the sample, and  $\pi^B$  is the average (net) growth rate of the price index for advertising expenses, which is 5.4% in the sample period used for the estimation.  $g^B$  is industry specific, and equal to the average growth rate of advertising expenses in that industry. Accordingly, we set  $g^B = 21.3\%$  for firms in the low-skill

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<sup>15</sup>Specifically, the price index for brand capital is the average yearly Producer Price Index by Industry: Advertising Agencies (U.S. Bureau of Labor Statistics, “Producer Price Index by Industry: Advertising Agencies” [PCU541810541810], retrieved from FRED, Federal Reserve Bank of St. Louis). Because this data series only starts in 1996, we extrapolate backwards using as predictors “Personal Consumption Expenditures: Chain-type Price Index, Index 2009=100 (BEA)”, “Gross Private Domestic Investment: Fixed Investment: Nonresidential: Intellectual Property Products (chain-type price index), Index 2009=100”, “Gross Private Domestic Investment: Fixed Investment: Nonresidential: Intellectual Property Products: Research and Development (chain-type price index), Index 2009=100 (BEA)”, “Gross Private Domestic Investment: Fixed Investment: Nonresidential: Intellectual Property Products: Entertainment, Literary, and Artistic Originals (chain-type price index), Index 2009=100 (BEA)”, and “Private fixed investment, chained price index: Nonresidential: Intellectual property products: Software, Index 2009=100 (BEA)” from the period of 1929 until 1995). (Note: the IPP software series only starts in 1959 so it only enters as a predictor after 1959).

industry,  $g^B = 22.2\%$  for firms in the mid-skill industry, and  $g^B = 34.7\%$  for firms in the high-skill industry. As in Vitorino (2014), we use a depreciation rate for brand capital of 20%. Once we have the initial capital stock, we iterate forward using the depreciation rate, advertising expenses, and investment price index. The investment rate on brand capital is then given by the ratio of the current period investment and the beginning of the period corresponding brand capital stock  $I_t^B/K_t^B$ .

Many firms do not report advertising expenditures. We treat the missing advertising expenditure data in a similar manner to the missing R&D expenditure data. We exclude from the sample the firms that never report advertising. For firms in which advertising expenditure data is missing for some, but not all, years and SG&A data is not missing, we impute the advertising expenditure data for the missing years based on the firm-level average advertising expenses to SG&A ratio. Again, the rationale for this procedure is the fact that SG&A data includes advertising expenses (together with other expenses). In robustness checks, we find that our main findings are robust to excluding firm-year observations with imputed advertising expenditures data, but using this imputation method allows us to expand the sample size.

*Additional firm-level variables:* We measure debt,  $B_{it+1}$ , as net total debt. Specifically, we measure net debt as long-term debt (item DLTT in Compustat) plus short term debt (item DLC), minus cash (item CHE), setting them to zero where they are missing. The market value of equity,  $P_{it}$ , is the closing price per share (item PRCC\_F) times the number of common shares outstanding (item CSHO). For firms with different fiscal year ends the price matches the firm’s fiscal year (and thus the timing of the accounting data).

We measure the tax rate,  $\tau_t$ , as the statutory corporate income tax for the highest bracket from the Commerce Clearing House, annual publications, until 2010, and from Deloitte after 2010. Stock variables subscripted  $t$  ( $t + 1$  for debt) are measured and recorded at the end of year  $t$ , while flow variables subscripted  $t$  are measured over the course of year  $t$  and recorded at the end of year  $t + 1$ .

*Labor skill industry classification:* Following Belo et al. (2017), we separate the economy into three broad industries based on the average labor skill of the workforce in the industry. Specifically, we classify an industry to be a low- medium, or high-skill industry based on the percentage of workers in that industry that work on occupations that require a high level of training and preparation (high-skill workers), using the Specific Vocational Preparation (SVP) index from the Dictionary of Occupational Titles (DOT), available from the Department of Labor, and employee data from the Bureau of Labor Statistics (BLS), Occupational Employment Statistics (OES) program. We use the data from Belo et al. (2017) to construct this industry classification (data available from the author’s webpage). The base industry-level data is available at the three-digit Standard Industry Classification (SIC) level before and including year 2001, and at the four-digit North American Industry Classification System (NAICS) after 2001. An industry is classified as a high labor-skill industry if it belongs to a 3-SIC or 4-NAICS industry in which the percentage of high-skill workers in that industry (variable PSKILL) is above the 70<sup>th</sup> percentile of the cross-sectional distribution (across industries) of the PSKILL variable. We classify an industry as a medium labor-skill industry if the percentage of high-skill workers in that industry is between the 30<sup>th</sup> and 70<sup>th</sup> percentile of the cross-sectional distribution of the PSKILL variable. Finally, we classify an industry as a low labor-skill industry if the percentage of high-skill workers in that industry is below the 30<sup>th</sup> percentile of the cross-sectional distribution of the PSKILL variable. Because the data only refers to the period from 1991 to 2013, for the period from 1975 to 1990, we used an average of the data from 1991 to 2001 and, for the period from 2014 to 2016, we used an average of the data from 2002 to 2013.

*Test assets:* As noted, the estimation is performed at the portfolio-level. Following Belo, Xue, and Zhang (2013), we consider portfolios sorted on firm’s lagged valuation ratio,  $VR_{i,t-1}$  (closely related to the standard market-to-book ratio). By construction, this portfolio sort generates a large cross-sectional spread in the valuation ratio across the portfolios, thus helping the identification of the model parameters. We sort all stocks in January of each year  $t$  into five portfolios based on the quantile of the firm’s valuation ratio cross sectional distribution for the fiscal year ending in  $t - 1$ .

The portfolios are rebalanced every year. In the robustness section, we examine the sensitivity of the results to the number of portfolios.

The final sample used for the estimation of the model includes data from 4,697 firms and 54,330 firm-year observations, for the period from 1975 to 2016. We start in 1975 because that is the year in which the Financial Accounting Standards Board (FASB) required firms to expense all R&D expenditures during the year in which the expenses were incurred. The following data items restricted the sample size as follows (percentage numbers correspond to sequential elimination of observations). First, as noted, we include only firms that report R&D expenses at least once during the firm’s lifetime.<sup>16</sup> This criteria eliminates 39% of the data. Next, 31% of observations are dropped due to missing physical capital investment rate or physical capital stock data.<sup>17</sup> Next, 7.2% of observations are dropped due to negative valuation ratio (because of the use of net debt, the total valuation ratio can be negative in some cases), 3.7% of observations are dropped due to missing hiring rate and number of employees data, 0.2% of the observations are dropped due to missing investment rate in knowledge capital or knowledge capital stock data, and 2% of the observations are dropped due to missing brand capital investment rate or brand capital stock data. Also, following Falato, Kadyrzhanova, and Sim (2014), 0.64% of observations are dropped due to missing sales data (we interpret observations with non-missing sales data as a proxy for data with higher quality). Finally, 3.3% of the observations are dropped due to missing portfolio-sorting variable (firm-level lagged valuation ratio).

As a robustness check, and to further evaluate the importance of the non-physical capital inputs for firm value, we estimate a version of the model without knowledge capital using the sample of

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<sup>16</sup>The large number of observations dropped due to missing R&D (and advertising expenditures) data is expected given that some firms do not report separately R&D or advertising expenses from SG&A data, and is thus a well known problem with using these data items from Compustat. As noted in Section 3.2, the estimation approach at the portfolio-level, not firm-level, mitigates some of the concerns with this large number of missing observations because the portfolio-level moments are likely to be more stable with respect to firm exit and entry or other accounting issues than firm-level moments, and hence the method is likely to be more robust to missing data.

<sup>17</sup>This large 31% figure is mostly driven by the fact that the first investment rate (investment scaled by beginning of the period capital stock) requires the lagged capital stock variable. Hence, the requirement of nonmissing investment rate forces us to drop the first observation of every firm in the sample.

firms with missing or zero knowledge capital that were excluded from the main sample. This alternative sample includes 2,964 firms, and 28,219 firm-year observations.

### 3.4 Summary Statistics

Panel A in Table 1 reports the summary statistics (time-series average of the cross sectional median, denoted as median, and standard deviation) of the valuation ratios and its components according to equation (21), across all firms in the economy, and separately in the low-, mid-, and high-skill industries.

The median valuation ratio across all firms is 1.97. This valuation ratio is higher in high labor-skill industries than in low labor-skill industries, 2.18 versus 1.51, respectively. Investment in knowledge capital has the highest median rate (26%), while investment in labor, the gross hiring rate, has the lowest median rate (17%). The investment and hiring rates are all higher in the high-skill industries than in the low-skill industries. In terms of volatility, the physical capital investment rate is the most volatile investment series, with a standard deviation of 79% per annum.

[Insert Table 1 here]

In terms of the average size of the scaled capital and labor inputs (scaled by assets, measured as the sum of the physical capital, knowledge capital, and brand capital stocks), the largest scaled input is the wage bill (using lagged wages as implied by equation (21)), which amounts to 63% of total assets. The second largest input is physical capital, with 46% of total assets. The ratio of knowledge capital stock to total assets is 31%. The smallest capital stock is brand capital with 11% of total assets. The relative magnitude of the ratios varies across the labor skill industries. For example, the scaled physical capital stock is higher in low-skill than in high-skill industries, 65% versus 36% of total assets, respectively. Similarly, the scaled brand capital stock is higher in low-skill than in high-skill industries, 20% versus 10% of total assets, respectively. Conversely, the scaled knowledge capital stock is lower in low-skill than in high-skill industries, 8% versus 16% of

total assets, respectively. Clearly, knowledge capital is more important in high-skill than in low-skill industries, while brand capital is more important in low-skill than in high-skill industries.

The shadow prices of the labor and capital inputs in equations (17) to (20) are determined by the investment/hiring rates. Thus, understanding the properties of the investment/hiring rates is useful for understanding the time-series properties of the value the inputs. Panel B in Table 1 reports the investment and hiring rate cross-correlations across all firms in the economy (the correlation patterns within industries is similar and hence, to save space, we do not report them here). The table shows that, as expected, the investment/hiring rates are all positively correlated. The correlations range from a minimum of 36% for the correlation between hiring and investment in knowledge capital, to 56% for the correlation between investment in knowledge and brand capital. These correlations are significantly smaller than one, thus suggesting that there is at least some independent variation in the shadow prices, and hence the market value, of the different capital and labor inputs.

## 4 Empirical Results

This section reports the main empirical findings. The analysis is performed across all firms and across labor skill industries. This industry classification (relative to other industry classifications available in the literature) is interesting for the purposes of our analysis because there are a priori reasons to expect that the adjustment cost parameters, and hence the importance of capital and labor inputs for firm value, vary in a systematic way across the different skill industries. First, as discussed in Belo et al. (2017) (also, see references therein) previous empirical studies find that it is more costly to replace a high-skill worker than a low-skill worker. Thus, this suggests that the labor adjustment costs parameters should differ across these industries, in particular, they should imply higher labor adjustment costs in high labor-skill industries. Also, all else equal, the higher labor adjustment costs imply that labor should represent a higher fraction of firm value in high labor-skill industries. Second, Belo et al. (2017) also provide evidence that investment in intangible capital

inputs such as R&D expenditures is higher in high labor-skill than in low labor-skill industries. Taken together, this suggests that the relative importance of the different capital and labor inputs for firm value should vary across industries with different skill levels.

Section 4.1 provides a firm value decomposition based on the book-value of the capital and labor inputs. Section 4.2 reports the model's estimation results across all firms in the economy and for low, medium, and high labor-skill industries. In addition, this section provides a comparison of the model fit relative to simplified versions of the model with fewer capital inputs, including the one-capital input model.

#### 4.1 Firm-Value Decomposition Based on Book Values

Before performing a formal estimation of the model, we can use the scaled capital input moments reported in Table 1 to make a preliminary assessment of the relative importance of each input for firm value based on the book-value of the inputs. If adjustment costs are zero, the shadow prices of the capital and labor inputs in equations (17) to (20) are simply 1, 0,  $(1 - \tau_t)$ , and  $(1 - \tau_t)$ . As a result, the value of each input is equal to its book-value, and the importance of each input for firm value can be computed from equations (12) to (15) without having to perform any estimation.

[Insert Table 2 here]

Table 2 reports a firms' book-value decomposition across all firms in the economy, and separately for the low-, med-, and high-skill industry. To obtain this decomposition, we evaluate equations (12) to (15) at the median value of the ratio of the (scaled) capital inputs, using the average tax rate in our sample of 38.1%. Without labor adjustment costs, as discussed in the Introduction section, the value of the installed labor force is zero. Across all firms, the most important input is physical capital, which represents about 62.4% of firms' book-value. The second most important input is knowledge capital which represents 27.6% of firm's book-value, and the least important input is brand capital which represents about 10% of firms' book-value. These numbers vary significantly across industries. The importance of physical capital and brand capital for the book-

value of the firm is significantly higher in the low-skill than in the high-skill industries, with 78.1% versus 49.7%, respectively for physical capital, and with 15.9% versus 8.8%, respectively for brand capital. Conversely, the value of knowledge capital is significantly lower in low-skill than in high-skill industries, with 5.9% versus 41.4%, respectively.

In the presence of adjustment costs, the shadow prices of each input vary over time. As a result, the relative importance of each input for a firm’s market value will be different from this baseline case.

## 4.2 Estimation

We first estimate the model using pooled data from all firms in the economy, thus assuming a homogeneous adjustment cost technology across firms. Next we allow for heterogeneity in the adjustment cost technology across industries and estimate the model separately across low-, medium-, and high-skill industries.

### Parameter Estimates and Model Fit

Column (1) of Panel A in Table 3, reports the point estimates of the adjustment cost parameters for all firms.

[Table 3 here]

The estimates of the slope adjustment cost parameters ( $\theta_i$ ) are  $\theta_P = 2.90$  for physical capital,  $\theta_L = 5.74$  for labor,  $\theta_K = 2.99$  for knowledge capital, and  $\theta_B = 2.21$  for brand capital. All the slope adjustment cost coefficients are statistically significant, which implies that we cannot reject the hypothesis that these inputs are subject to adjustment costs.

The estimates of the curvature adjustment cost parameters ( $\nu_i$ ) are  $\nu_P = 2.43$  for physical capital,  $\nu_L = 1.68$  for labor,  $\nu_K = 1.27$  for knowledge capital, and  $\nu_B = 2.35$  for brand capital. The evidence thus suggest that for labor and knowledge capital, the adjustment cost function has less curvature than the standard quadratic adjustment cost specification which, for tractability, is

often used in the investment literature. For capital, the estimates suggest that the adjustment cost function has slightly more curvature than the quadratic adjustment cost specification. This result is consistent with the findings in BXZ who estimate a curvature parameter for the physical capital adjustment cost function that is significantly higher than 2. Finally, the evidence implies that the firm's optimization problem has an interior solution because the point estimates of both the physical capital, labor, and the two intangible capital adjustment cost parameters are consistent with the adjustment cost function being increasing and convex in the investment/hiring rates.

Turning to the analysis of the model fit, column (1) of Panel B in Table 3 provides four measures of fit. Specifically, the table reports: i) the cross sectional  $R^2$  (denoted XS- $R^2$ ) of a scatter plot of the average portfolio-level valuation ratio against the average portfolio-level predicted (model-implied) valuation ratio; ii) the time series  $R^2$  measure of the pooled portfolio-level data (which is the measure that is implicitly targeted in the estimation); iii) the mean absolute errors (m.a.e.), computed as the means of the absolute errors of the error term of each portfolios; and iv) the m.a.e. as a fraction of the average valuation ratio of each portfolio (thus providing a relative measure of the size of the error term across portfolios).

According to the four metrics considered here, the model performs well. The time series  $R^2$  is 80%. In addition, the cross sectional  $R^2$  is quite high, 99%, even though the model estimation does not explicitly target the cross sectional  $R^2$  moment. Thus, the model performs well both in the cross sectional and in the time series dimensions. In terms of average valuation ratio errors, the model mean absolute error (m.a.e.) is quite low, at only about 26% of the mean portfolio-level valuation ratio.

Figure 1 provides a visual description of the good fit of the model. Panel A shows the time series plot of the cross-sectional average (across the 5 portfolios used as test assets) portfolio median valuation ratio observed in the data (realized VR) and predicted by the model (predicted VR). Panel B shows the scatter plot across portfolios of the time series average of the cross-sectional median valuation ratio observed in the data against the value predicted by the model.

[Figure 1 here]

Panel A in Table 3, columns (2) to (4), report the point estimates of the adjustment cost parameters in the low (L), medium (M), and high (H), labor-skill industries, respectively. The estimate of the slope adjustment cost parameter for labor increases with the average labor-skill level of the industry, from  $\theta_L = 3.75$  in the low-skill industries to  $\theta_L = 5.75$  in the high-skill industries. Similarly, the slope adjustment cost parameter for knowledge capital is higher in high-skill industries than in low-skill industries,  $\theta_K = 0.66$  (and statistically insignificant) versus  $\theta_K = 3.94$ , although the relationship is not monotone (its value is higher in mid-skill industries). Going in the opposite direction, the slope adjustment cost parameters for physical capital and brand capital in general decrease with the average labor-skill level of the industry. Turning to the analysis of the curvature adjustment cost parameters, we note that almost all point estimates satisfy the theoretical restrictions ( $v_i > 1$ ), but the variation in the point estimates across industries is not monotone.

Turning to the analysis of the model fit, Panel B in Table 3, columns (2) to (4), reports the four metrics of the model fit in each labor-skill industry. The model performs particularly well in explaining the cross-sectional and time-series variation in the high-skill industries, with a cross sectional  $R^2$  of 98%, and a time-series  $R^2$  of 80%. In terms of average valuation ratio, the model mean absolute error in the high-skill industries is only 27% of the mean valuation ratio in those industries. The model fit in low-skill industries is more modest but still reasonable, with a cross sectional  $R^2$  of 96%, and a time-series  $R^2$  of 65%. In terms of average valuation ratio, the model mean absolute error in the low-skill industries is about 36% of the mean valuation ratio in the low-skill industries. Figure 2 provides a visual description of the fit of the model in each of the three-skill industries, both in the time-series (TS) and in the cross-section (XS).

[Figure 2 here]

## Firm-Value Decomposition and Adjustment Costs

The parameter estimates allow us to infer the contribution of each capital and labor input for the firm's market value, and quantify the magnitude of the adjustment costs of each input. Thus, in this section, we provide an economic interpretation of the parameter estimates in terms of their implications for firm's market value decomposition and implied magnitude of adjustment costs.

To obtain the implications of the parameter estimates for the firm's market value, we use the estimates reported in Panel A of Table 3 to compute, for each firm and in each year, the values of  $q_{it}^P \frac{K_{it+1}^P}{A_{it+1}}$ ,  $q_{it}^L \frac{L_{it+1}}{A_{it+1}}$ ,  $q_{it}^K \frac{K_{it+1}^K}{A_{it+1}}$ , and  $q_{it}^B \frac{K_{it+1}^B}{A_{it+1}}$ , that is, the scaled value of each capital/labor input. We then compute the cross sectional median value of the previous values and substitute these values in equations (12) to (15) to compute, in each year, the fraction of the firm value attributed to each capital/labor input. We interpret this procedure as capturing the firm market-value decomposition for the median firm in the economy.<sup>18</sup>

Panel B in Table 3 (column (1), firm value decomposition), reports the time-series average of the previous fraction of the firm value attributed to each capital/labor input for all firms. The four inputs are important determinants of firm value. In the baseline specification, physical capital accounts for 25.1% of firms' market value, the installed labor force accounts for 49.1%, knowledge capital accounts for 21.9%, and brand capital accounts for the remaining 4.0%. This analysis reveals that physical capital accounts for only a quarter of the firm's total market value on average. Clearly, in the modern economy, intangible capital and labor are the most important determinants of firm value.

Next, we perform the same analysis across labor-skill industries. The estimation results show that the relative importance of the capital/labor inputs exhibits substantial variation across the different labor-skill industries. Confirming the importance of labor, especially in the high-skill industries, Panel B in Table 3, columns (2) to (4) shows that the average fraction of firm value

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<sup>18</sup>Alternatively, one could compute the fraction of firm value attributed to each capital/labor input for each firm in the economy, take the cross-sectional median of these values, and report the time series average of this median. This procedure does not work here because the sum of the cross sectional median weights does not add up to one.

attributed to labor increases with the average labor-skill level of the industry. In the low-skill industries, the fraction of firm value that can be attributed to labor is on average 18.2%, whereas in the high-skill industries this fraction is 40.8%. Similarly, the fraction of firm value attributed to knowledge capital also increases with the average labor-skill level of the industry. In the low-skill industries, the fraction of firm value that can be attributed to knowledge capital is on average only 0.9%, whereas in the high-skill industries this fraction is 33%. Going in the opposite direction, the fraction of firm value attributed to physical capital and brand capital decreases with the average labor-skill level of the industry. In the low-skill industries, the fraction of firm value that can be attributed to physical capital is on average 57%, whereas in the high-skill industries this fraction drops to 22.7%. Similarly, in the low-skill industries, the fraction of firm value that can be attributed to brand capital is on average 24%, whereas in the high-skill industries this fraction drops to 3.5%.

What explains the relatively high importance of labor and intangible capital inputs, in addition to physical capital, for firm value? For the intangible capital inputs, part of the value comes from the book value of the capital stocks as noted in Table 2. That is, even without adjustment costs, the intangible capital inputs contribute in a non-trivial way to the firm's market value due to the size of the capital stocks. With adjustment costs, the relative importance of the inputs changes because the adjustment costs affect the shadow prices of the capital stocks. In particular, the contribution of labor for firm value depends crucially on to existence of positive labor adjustment costs. Thus, to understand the firm value decomposition estimates, here we evaluate the economic magnitude of the adjustment costs of the four inputs. Naturally, when an input is costly to adjust, the installed values of the inputs are valuable to the firm because they contribute not only for production but also allow the firm to avoid adjustment costs in the future.

Panel B in Table 3 (adjustment costs), reports the implied proportions of firms' sales that are lost due to physical capital, labor, and intangible capital adjustment costs. Using the functional form specification in equation (16), these values are computed as a fraction of firms' total sales  $Y_{it}$

as follows:

$$\frac{CP_{it}}{Y_{it}} = \frac{\frac{1}{\nu_P} \left| \theta_P \frac{I_{it}^P}{K_{it}^P} \right|^{\nu_P} K_{it}^P}{Y_{it}} \quad (26)$$

$$\frac{CL_{it}}{Y_{it}} = \frac{\frac{1}{\nu_L} \left| \theta_L \frac{H_{it}}{N_{it}} \right|^{\nu_L} W_{it} L_{it}}{Y_{it}} \quad (27)$$

$$\frac{CK_{it}^K}{Y_{it}} = \frac{\frac{1}{\nu_U} \left| \theta_K \frac{I_{it}^K}{U_{it}^K} \right|^{\nu_K} U_{it}^K}{Y_{it}} \quad (28)$$

$$\frac{CK_{it}^B}{Y_{it}} = \frac{\frac{1}{\nu_B} \left| \theta_B \frac{I_{it}^B}{U_{it}^B} \right|^{\nu_B} U_{it}^B}{Y_{it}}. \quad (29)$$

We compute the adjustment costs estimates in an analogous way to the computation of the fractions of firm value. Specifically, we first compute the value in equations (26) to (29) for each firm and in each year. Then, in each year, we compute the cross sectional median of the previous values, and report the time-series average of these medians.

Results in column (1) show that for all firms, the estimated magnitude of labor and, to a lesser extent, knowledge capital adjustment costs is large, whereas the magnitudes of physical capital and brand capital adjustment costs is more modest. On average, the fraction of (annual) sales that is lost due to labor adjustment costs is 22.3%. The fraction of sales that is lost due to knowledge capital adjustment costs is 9.1%, and for brand capital this fraction is 0.5%. The fraction of sales that is lost due to physical capital adjustment costs is estimated to be low, 3.8%. Although there is no consensus on the magnitude of labor and capital adjustment costs, the estimated values of adjustment costs for these two inputs are within the empirical estimates surveyed in Hamermesh and Pfann (1996), and discussed in Merz and Yashiv (2007). For brand capital, the estimated value of adjustment costs is lower than those estimated in Vitorino (2014) (on average, about 8% of firm's annual sales). The difference is that we are estimating firm-level parameters whereas Vitorino (2014) estimates portfolio-level parameters. In addition, we consider a model with four inputs whereas Vitorino (2014) only considers physical capital and brand capital.

Turning to the analysis of the variation in the size of adjustment costs across industries, Panel

B in Table 3, columns (2) to (4) shows that the estimated labor and knowledge capital adjustment costs increase significantly with the average labor-skill level of the industry. The fraction of (annual) sales lost due to labor adjustment costs is on average 10.3% in the low-skill industries, and 26.9% in the high-skill industries. Similarly, the fraction of (annual) sales lost due to knowledge capital adjustment costs is on average 0% in the low-skill industries, and 21% in the high-skill industries. This positive relationship between the size of adjustment costs and the average labor-skill of the industry is reversed for the physical capital and brand capital inputs (although the relationship is not monotone for physical capital inputs). The fraction of (annual) sales lost due to brand capital adjustment costs is on average 13.6% in the low-skill industries, and 1.6% in the high-skill industries. The fraction of sales lost due to physical capital adjustment costs is on average 18.4% in the low-skill industries, and 11.2% in the high-skill industries.

Taken together, these point estimates show that labor is the input that is subject to the highest adjustment costs. This finding helps understand why the value of the firms' installed labor force is such an important component of firms' market value, and why the firm value decomposition based on the real shadow prices of the capital and labor inputs differs significantly from the firm value decomposition based on the book value of the inputs reported in Table 2.

### **Model Comparison**

To help understand the fit of the model and the relative importance of each capital input for firm valuation, we estimate restricted versions of the model using different subsets of the inputs (for the alternative model specifications, we only report the estimation results in the low and high-skill industries, and omit the results for the mid-skill industry to save space). Results are presented in columns (5) and (12) of Table 3. Note that the only parameter estimates that violate the theoretical restrictions are the intangible capital curvature parameters for the models that include physical capital and either knowledge capital or brand capital in the low-skill industries, reported in columns (9) and (11). Here, the estimates of the curvature parameters of the knowledge capital

and brand capital adjustment costs are both less than 1, suggesting that a model with only physical capital and either knowledge capital or brand capital, is a misspecified model for these industries.

Comparing across model specifications, Panel B in Table 3 shows that the contribution of each input for the improvement of the model fit varies across industries. Adding labor or knowledge capital to the one-capital input model has a first order and similar impact on the quality of the model fit in high-skill industries, whereas adding brand capital has a first order impact on the quality of the model fit in low-skill industries. For tractability, we focus our discussion here on the time-series  $R^2$ , because this metric is the most informative for this analysis due to its higher variation across model specifications. For example, comparing columns (6) and (8), the time-series  $R^2$  in the high-skill industries increases from 62% to 75% when quasi-fixed labor is added to the one-capital input model. Comparing columns (5) and (7), the time-series  $R^2$  in the low-skill industries increases only slightly from 53% to 57% when quasi-fixed labor is added to the one-capital input model (the analysis regarding the incremental contribution of knowledge capital is similar). Different from the previous inputs, the improvement from adding brand capital to the one-capital input model is more concentrated in the low labor-skill industries. Comparing columns (6) and (12), the time-series  $R^2$  in high-skill industries remains basically unchanged (62% and 60%, respectively) when brand capital is added to the one-capital input model. Comparing columns (5) and (11), the time-series  $R^2$  in low-skill industries increases from 53% to 61% when brand capital is added to the one-capital input model.

Taken together, the results from the estimation of the model across industries reported in the previous subsections can be summarized as follows. First, allowing for technology heterogeneity across industries seems important for a proper characterization of the importance of the capital and labor inputs for firm value. Second, adding additional inputs to the baseline one-capital input model is especially important in high-skill industries. While in low-skill industries, the value of physical capital represents about 57% of firm's market value, in high-skill industries the value of physical capital represents less than 25% of firm's market value. Thus, in high-skill industries,

the majority of the firm's market value can be attributed to inputs other than physical capital, namely, labor, knowledge capital, and, to a lesser extent, brand capital. Finally, the analysis shows that, while intangible capital is important in both high- and low-skill industries, the type of intangible capital that is the most important depends on the industry. In low-skill industries the contribution of brand capital for firm value is significantly higher than the contribution of knowledge capital (23.4% vs 0.9%, respectively), but this pattern is reversed in high-skill industries (3.54% vs 33%, respectively). These results highlight the importance of considering heterogeneous measures of intangible capital in empirical work.

## **5 Time-Series and Risk Characteristics of Labor and Capital Inputs**

In this section we use the parameter estimates obtained in the previous section to perform additional analyses. First, we evaluate if the importance of each capital/labor input for firm value changed over time. Second, we evaluate the business cycle properties of the value of each capital/labor input. This last analysis is useful because it allows us to understand the risk properties of each capital/labor input, and hence the risk properties of firm market value.

### **5.1 Value Decomposition Across Decades**

The analysis in the previous section reports the time-series averages of the firm value decomposition in the full sample from 1975 to 2016. To provide a more detailed characterization of the data, here we perform the same analysis across different sub-periods (we do not re-estimate the parameters' values because the model assumes they are constant over time). We perform the analysis using the estimates obtained using all firms in the economy and also the estimates obtained from separately estimating the model for different labor-skill industries. To compute the fraction of firm value attributed to each capital input across all firms we use the estimates from column (1), Panel A in Table ???. For the analysis across labor-skill industries, we use the estimates from columns (1) to (3), Panel A, in Table 3.

[Table 4 here]

Table 4 reports the time series averages of the fraction of firm value attributed to each capital inputs across decades: 1970s (1975-1979), 1980s (1980-1989), 1990s (1990-1999), 2000s (2000-2009), and 2010s (2010-2016). Figure 3 provides a visual description of the trends in the input value shares in the data, both across all firms, and in the low-, mid-, and high-skill industries.

[Figure 3 here]

Across all firms, the table and the figure allow us to identify interesting patterns in the data. First, the importance of knowledge capital has increased over our sample period from 9.3% in the 1970s to 28.7% in the 2010s. The increased importance of knowledge capital has crowded out the importance of the labor input and, more significantly, of physical capital. Over the sample period, the importance of the labor input has slightly decreased from 55.2% in the 1970s to 47.9% in the 2010s (although this decrease is not monotone over time). The slight decrease in the contribution of labor for firm value resembles the well documented decrease of the aggregate labor share over the same sample period, as noted in Elsby, Hobijn, and Şahin (2013), Karabarbounis and Neiman (2013), among others. At the same time, the importance of physical capital for firm value has decreased significantly from 30.9% in the 1970s to 20.7% in the 2010s. The contribution of brand capital for firm value, when evaluated across all firms, is relatively small during the entire period, but it has decreased slightly in recent years.

Turning to the analysis of the change in the importance of each input for firm's market value across labor skill industries, Table 4 allows us to identify interesting differences across industries. Even though the main trends observed across all firms are pervasive across all industries, the trends are significantly more pronounced in the mid- and, especially, in the high-skill industries. For example, in the high-skill industries, the importance of knowledge capital for firm value has more than doubled, increasing from 16% in the 1970s to 35.2% in the 2010s, while the importance of physical capital has halved, decreasing from 32.1% in the 1970s to 16.2% in the 2010s. Also, the

slight decrease in the importance of brand capital for firm value is concentrated in the mid and high-labor skill industries, only. In the low labor-skill industry, the importance of brand capital for firm value has increased from 17.6% in the 1970s to 29.2% in the 2010s.

Taken together, the analysis in this section further highlights the importance of the non-physical capital inputs for understanding firm value, especially in the most recent decades, and in high-skill industries, in which the non-physical capital inputs account for, on average, about 84% of firm value. The importance of physical capital for firm value is significantly lower in recent years when compared to the earlier part of the sample, while the importance of intangible capital, broadly defined, is significantly higher. But the type of intangible capital that has gained importance in recent years varies across industries. In mid- and high-skill industries, there is a significant increase in the importance of knowledge capital, with only small changes in the importance of brand capital. In low-skill industries, there is a significant increase in the importance of brand capital, with only small changes in the importance of knowledge capital. Finally, the compositional change in the importance of each input for firm value highlights the importance of targeting the time series of the valuation ratios in the estimation, as opposed to only targeting the cross sectional time-series means of the valuation ratios, as in LWZ/BXZ.

## 5.2 Risk Characteristics of Labor and Capital Inputs

In addition to the analysis of the contribution of each input for firm value, the parameter estimates allow us to characterize the business-cycle properties of the value, and corresponding firm value shares, of each input. This analysis is useful because it allows us to understand the risk characteristics of the inputs, and hence the risk characteristics of the firm.

We proceed as follows. Given the parameters estimates, we compute for each firm the time series of the firm's model-implied valuation ratio, as well as the time series of the (scaled) value of each capital input. These series are given by:

$$\begin{aligned}
V_{it}^P &= \hat{q}_{it}^P \frac{K_{it+1}^P}{A_t} : \text{value of physical capital} \\
V_{it}^L &= \hat{q}_{it}^L \frac{L_{it+1}}{A_t} : \text{value of labor} \\
V_{it}^K &= \hat{q}_{it}^K \frac{U_{it+1}^K}{A_t} : \text{value of knowledge capital} \\
V_{it}^B &= \hat{q}_{it}^B \frac{U_{it+1}^B}{A_t} : \text{value of brand capital} \\
VR_{it} &= V_{it}^P + V_{it}^L + V_{it}^K + V_{it}^B : \text{valuation ratio,}
\end{aligned}$$

in which the hat on top of the marginal  $q$ 's denotes estimated value. Then, in each year, and consistent with the approach in the previous sections, we compute the cross-sectional median of each component, and also of the firm's valuation ratio (VR). We then use these values to compute the share of each input for firm value. Because we are interested in understanding the business cycle properties of these components, we then extract the cyclical component of the log of the previous variables through an HP filter (with a smoothing factor of 100). The cyclical components are measured in percentage deviation relative to the trend. We also extract the cyclical component of aggregate sales using a similar procedure. To understand the volatility and the cyclicality of the input shares and values, we then compute the covariance of the cycle component of each input share with the business cycle, measured by the cycle in aggregate sales ( $Y^{\text{agg}}$ ). We compute the moments across all firms in the economy, and also separately for low-, mid- and high-skill industries.

[Table 5 here]

Table 5 reports the results from this analysis. Panel A shows that, as expected, the valuation ratio is procyclical, especially in the high-skill industries. The analysis of the covariance of the cyclical components of capital and labor firm-value shares with aggregate sales across all firms reveals an interesting pattern. While the share of labor in firm value is procyclical (positive correlation with aggregate sales), the shares of the capital inputs are countercyclical. The analysis across industries shows that this pattern is mostly driven by the firms in the high labor-skill

industries. Thus, the importance of the labor input for firm's market value is relatively higher during good economic times.

Turning to the analysis of the cyclical components of the scaled value of capital/labor inputs, Table 5 shows that, across all firms, the value of physical capital, labor, and brand capital are procyclical, whereas the value of knowledge capital is countercyclical (which explains why the share of knowledge capital in firm value is countercyclical). In addition, the scaled value of labor is the most volatile of all of the input values. The standard deviation of the cyclical component of the scaled value of the labor input is 0.26, which makes this component about three times more volatile than the cyclical component of the scaled value of the other inputs (all below 0.09 across all firms). These figures help understand the pattern of the cyclicity of the capital/labor input shares in firm value. In good times, the share of labor in firm value increases because the value of labor is procyclical, and this increase is relatively higher than that of the other inputs due to the higher volatility of the scaled value of labor (that is, the covariance – correlation times standard deviations – of the scaled value of this input with aggregate sales is the highest among all the inputs). Together with the fact that labor has the largest weight on firm value (across all firms, the weight of labor on firm value is about 50% on average), the previous two features combined make the share of labor on firm value procyclical. The relatively larger increase in the scaled value of labor in good times also explains why the share of physical capital and the share of brand capital in firm value actually decrease in good times, despite the increase in the value of these inputs in good times.

Taken together, our analysis shows that the value of labor is the most procyclical (as measured by the covariance of the scaled value of labor with aggregate sales) and volatile component of firm value. Thus, understanding the dynamics of labor inputs (and their associated labor market frictions) across firms is important for understanding the dynamics of firm value.

## 6 Robustness

To check the robustness of our main findings, in particular, the importance of non-physical inputs for understanding firm value, we re-estimate the parameters of the model across several perturbations of the empirical procedures. Specifically, we re-estimate the model parameters using a different number of portfolios as test assets, and using other portfolio-level moments, not just the cross sectional median. We also report the results from firm-level estimation. Finally, we re-estimate a restricted version of the model using a different sample which includes only firms that do not report R&D expenses. To facilitate the analysis, and avoid a proliferation of tables, we estimate the model parameters using the pooled sample of all firms in the economy.

### 6.1 Different Number of Portfolios

Panel A in Table 6, columns (2) to (4), reports the estimation results using a different number of portfolios as test assets. In the baseline estimation, we use 5 portfolios (sorted on firm's lagged valuation ratio). In column (2) we consider 2 portfolios, in column (3) we consider 10 portfolios, and in column (4) we consider 20 portfolios.

[Insert Table 6 here]

The point estimates reported in columns (2) to (4) appear to be similar in magnitude to the point estimates in the baseline case, reported in column (1). But it is difficult to judge the degree of similarity of the estimates based on these point estimates only. To help the interpretation of the results, it is useful to focus our analysis on the differences between the fractions of firm value implied by each set of point estimates. As reported in Panel B in Table 6, the main conclusion regarding the importance of the alternative (non-physical capital) inputs for firm value holds in these alternative specifications. We note, however, that the contribution of labor is somewhat lower (and, conversely, the contribution of physical capital and knowledge capital are somewhat higher) in columns (2) and (4) than in the baseline specification (1).

## 6.2 Different Target Moments

The baseline estimation of the model matches the behavior of the median firm in each portfolio. As discussed in Section 3.2, we also estimate the model by targeting a different set of cross sectional moments. In Table 6, column (5), we report the model estimates when we target the portfolio-level cross sectional equal-weighted average (XSEW), instead of the cross sectional median (XSMED) used in the baseline estimation. In addition to this method, in columns (6) and (7) we target alternative portfolio-level cross-sectional moments. In column (6), the estimation targets the portfolio-level inter-quartile valuation ratio spread ( $VR_{75-25}$ ), and in column (7), the estimation targets not only the cross sectional median ( $VR_{50}$ ) but also the 25<sup>th</sup> and 75<sup>th</sup> percentiles of the portfolio-level cross-sectional distribution of valuation ratios. Finally, in column (8) we drop the portfolio-level approach completely, and estimate the model parameters using firm-level data.

One technical issue arises in the estimation of the model at the firm-level or when we target the portfolio-level cross sectional average of firms' valuation ratios. These estimation approaches are very sensitive to outliers in the data, in contrast with the baseline estimation approach which targets the cross sectional median. So, in the results reported in columns (5) and (8), we use data winsorized at the top and bottom (if the variable admits negative values) 2% of the distribution of all the ratios included in the estimation. Recall that in the baseline estimation the data is not winsorized. That is one reason why we adopt the cross-sectional median estimation method as the primary estimation method.

The point estimates reported in Panel A of Table 6, columns (5) to (8), appear to be similar in magnitude to the point estimates in the baseline case, reported in column (1). Again, to help the interpretation of the results, it is useful to focus our analysis on the differences between the fractions of firm value implied by each set of point estimates. As reported in Panel B in Table 6, the contribution of each input for firm value is stable across columns (5) to (7). In column (8), using the firm-level estimation, the fraction of firm value attributed to labor is significantly lower than in the baseline case: 10.7% vs 49.9% in the baseline case. This result is expected if there

is substantial measurement error in firm-level labor data: in this case, we expect the adjustment cost parameter estimates to be biased towards zero, and hence the firm-value decomposition to be closer to the firm book-value decomposition discussed in Section 4.1.

### 6.3 Different Sample

As discussed in Section 3.3, in the main sample, we drop firms that never report (or always report zero) R&D expenses. To establish the importance of the non-physical capital inputs for understanding firm value, here we consider the sample that only includes the non- (or missing-) R&D firms that were excluded from the main sample. We then estimate a (restricted) version of the model with physical capital, labor, and brand capital only, thus excluding knowledge capital.

To save space, we present the full results from this analysis in the online appendix, and only provide here a summary of the main findings. The estimation results using this alternative sample of non R&D firms provides additional support for a model with multi-capital/labor inputs. Across industries, the average contribution of labor for firm value ranges from 42.9% (low-skill) to 52.2% (high-skill), whereas the average contribution of brand capital for firm value ranges from 6.5% (low-skill) to 14.7% (mid-skill). Thus, as in the main sample, the contribution of the non-physical capital inputs for firm value is substantial, accounting for more than 50% of firms' market value. The model fit is also good. The time series  $R^2$  are high and comparable with the baseline model. The times series  $R^2$  ranges from 67% (low-skill) to 87% (high-skill). These value are significantly higher than those for the one capital-good model, especially in the high-skill industries in which the time series  $R^2$  is 61%, that is, 26 percentage points lower than in the baseline model with multi-capital inputs.

Taken together, the estimation results using the alternative sample of non R&D firms, and the results from the previous subsections, show that the importance of the non-physical capital inputs for firm value appears to be a finding that is robust to reasonable variations of the empirical procedures, thus providing additional empirical support for models with multiple capital inputs as

main sources of firm value.

## 7 Conclusion

We incorporate quasi-fixed labor, knowledge capital, and brand capital into the neoclassical model of investment, and estimate the contribution of each input for explaining firm market value. The structural model performs well in explaining both the cross-sectional and the time-series variation of firms' market value across all firms, with a time series  $R^2$  of 80% and a cross sectional  $R^2$  of 99%. In addition, we find that the relative importance of each input for firm value varies across industries. On average, physical capital accounts for 22.7% to 56.7% of firms' market value across industries, installed labor force accounts for 18.2% to 40.1%, knowledge capital accounts for 0.9% to 33%, and brand capital for 3.5% to 24%. We show that financial markets assign large and positive values to the installed stocks of the different types of inputs because they are costly to adjust, allowing firms to extract some rents as a compensation for the cost of adjusting the inputs. Overall, our firm value decomposition provides direct empirical evidence supporting models with multiple capital inputs as main sources of firm value.

Our estimation results also allow us to characterize the time-series and business cycle properties of the market value of the different capital inputs. We document that the importance of physical capital has decreased substantially over the last four decades, while the importance of knowledge capital input has increased significantly, especially in high-skill industries (high tech sector). We also find that the value of labor is the most volatile and procyclical component of firm value, which suggest that understanding the dynamics of firms's labor inputs (and their associated labor market frictions) is useful for understanding the dynamics of firm values.

Finally, methodologically, our estimation procedure targets portfolio-level cross-sectional moments that allow us to estimate firm-level structural parameters and avoid the aggregation bias of the BXZ/LWZ estimation procedure. This is useful for practical applications because it allows us to compute market values at the firm- not portfolio-level, which is naturally more useful

in practice. Possible applications include the valuation of private firms or initial public offerings, guidance in merger and acquisition transactions, among other applications that require estimates of firm values.

## Appendix

### A Derivation: Firm Value Decomposition

The first order conditions with respect to  $I_{it}^P$ ,  $K_{it+1}^P$ ,  $H_{it}$ ,  $L_{it+1}$ ,  $I_{it}^K$ ,  $U_{it+1}^K$ ,  $I_{it}^B$ ,  $U_{it+1}^B$ , and  $B_{it+1}$ , from maximizing the cum-dividend market value of equity are:

$$q_{it}^P = 1 + (1 - \tau_t) \frac{\partial C_{it}}{\partial I_{it}^P} \quad (\text{A.1})$$

$$q_{it}^P = E_t \left[ M_{t+1} \left[ (1 - \tau_{t+1}) \left( \frac{\partial \Pi_{it+1}}{\partial K_{it+1}^P} - \frac{\partial C_{it+1}}{\partial K_{it+1}^P} \right) + \delta_{it+1}^P \tau_{t+1} + (1 - \delta_{it+1}^P) q_{it+1}^P \right] \right] \quad (\text{A.2})$$

$$q_{it}^L = (1 - \tau_t) \frac{\partial C_{it}}{\partial H_{it}} \quad (\text{A.3})$$

$$q_{it}^L = E_t \left[ M_{t+1} \left[ (1 - \tau_{t+1}) \left( \frac{\partial \Pi_{it+1}}{\partial L_{it+1}} - \frac{\partial C_{it+1}}{\partial L_{it+1}} - W_{it+1} \right) + (1 - \delta_{it+1}^L) q_{it+1}^L \right] \right] \quad (\text{A.4})$$

$$q_{it}^K = (1 - \tau_t) \left[ 1 + \frac{\partial C_{it}}{\partial I_{it}^K} \right] \quad (\text{A.5})$$

$$q_{it}^K = E_t \left[ M_{t+1} \left[ (1 - \tau_{t+1}) \left( \frac{\partial \Pi_{it+1}}{\partial U_{it+1}^K} - \frac{\partial C_{it+1}}{\partial U_{it+1}^K} \right) + (1 - \delta_{it+1}^K) q_{it+1}^K \right] \right] \quad (\text{A.6})$$

$$q_{it}^B = (1 - \tau_t) \left[ 1 + \frac{\partial C_{it}}{\partial I_{it}^B} \right] \quad (\text{A.7})$$

$$q_{it}^B = E_t \left[ M_{t+1} \left[ (1 - \tau_{t+1}) \left( \frac{\partial \Pi_{it+1}}{\partial U_{it+1}^B} - \frac{\partial C_{it+1}}{\partial U_{it+1}^B} \right) + (1 - \delta_{it+1}^B) q_{it+1}^B \right] \right] \quad (\text{A.8})$$

$$1 = E_t \left[ M_{t+1} \left[ r_{it+1}^B - (r_{it+1}^B - 1) \tau_{t+1} \right] \right] = E_t \left[ M_{t+1} r_{it+1}^{Ba} \right] \quad (\text{A.9})$$

In the last equation we define the after-tax bond return as  $r_{it+1}^{Ba} \equiv r_{it+1}^B - (r_{it+1}^B - 1) \tau_{t+1}$ .

Using the FOCs (A.2, A.4, A.6, and A.8),

$$\begin{aligned} & q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K U_{it+1}^K + q_{it}^B U_{it+1}^B \\ = & E_t \left[ M_{t+1} \left[ (1 - \tau_{t+1}) \left( \frac{\partial \Pi_{it+1}}{\partial K_{it+1}^P} K_{it+1}^P + \frac{\partial \Pi_{it+1}}{\partial L_{it+1}} L_{it+1} + \frac{\partial \Pi_{it+1}}{\partial U_{it+1}^K} U_{it+1}^K + \frac{\partial \Pi_{it+1}}{\partial U_{it+1}^B} U_{it+1}^B \right) \right. \right. \\ & - (1 - \tau_{t+1}) \left( \frac{\partial C_{it+1}}{\partial K_{it+1}^P} K_{it+1}^P + \frac{\partial C_{it+1}}{\partial L_{it+1}} L_{it+1} + \frac{\partial C_{it+1}}{\partial U_{it+1}^K} U_{it+1}^K + \frac{\partial C_{it+1}}{\partial U_{it+1}^B} U_{it+1}^B \right) \\ & + (1 - \delta_{it+1}^P) q_{it+1}^P K_{it+1}^P + (1 - \delta_{it+1}^L) q_{it+1}^L L_{it+1} + (1 - \delta_{it+1}^K) q_{it+1}^K U_{it+1}^K + (1 - \delta_{it+1}^B) q_{it+1}^B U_{it+1}^B \\ & \left. \left. + \delta_{it+1}^P \tau_{t+1} K_{it+1}^P - (1 - \tau_{t+1}) W_{it+1} L_{it+1} \right] \right]. \end{aligned}$$

With constant return to scale production and adjustment costs,

$$\begin{aligned}
& q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K U_{it+1}^K + q_{it}^B U_{it+1}^B \\
= & E_t [M_{t+1} [(1 - \tau_{t+1})(\Pi_{it+1} - C_{it+1} - I_{it+1}^K - I_{it+1}^B - W_{it+1}N_{it+1}) - I_{it+1}^P + \delta_{it+1}^P \tau_{t+1} K_{it+1}^P \\
& + (1 - \tau_{t+1}) \frac{\partial C_{it+1}}{\partial I_{it+1}^P} I_{it+1}^P + I_{it+1}^P + (1 - \tau_{t+1}) \frac{\partial C_{it+1}}{\partial H_{it+1}} H_{it+1} + (1 - \tau_{t+1}) \frac{\partial C_{it+1}}{\partial I_{it+1}^K} I_{it+1}^K \\
& + I_{it+1}^K + (1 - \tau_{t+1}) \frac{\partial C_{it+1}}{\partial I_{it+1}^B} I_{it+1}^B + I_{it+1}^B \quad (A.10) \\
& + (1 - \delta_{it+1}^P) q_{it+1}^P K_{it+1}^P + (1 - \delta_{it+1}^L) q_{it+1}^L L_{it+1} + (1 - \delta_{it+1}^K) q_{it+1}^K U_{it+1}^K \\
& + (1 - \delta_{it+1}^B) q_{it+1}^B U_{it+1}^B ] \quad (A.11)
\end{aligned}$$

$$\begin{aligned}
= & E_t [M_{t+1} [(1 - \tau_{t+1})(\Pi_{it+1} - C_{it+1} - I_{it+1}^K - I_{it+1}^B - W_{it+1}N_{it+1}) - I_{it+1}^P + \delta_{it+1}^P \tau_{t+1} K_{it+1}^P + B_{it+2} - r_{it+1}^B B_{it+1} \\
& + q_{it+1}^P K_{it+2}^P + q_{it+1}^L L_{it+2} + q_{it+1}^K U_{it+2}^K + q_{it+1}^B U_{it+2}^B - B_{it+2} ] + E_t [M_{t+1} r_{it+1}^{Ba} ] B_{it+1}.
\end{aligned}$$

Rearranging the above equation,

$$q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K U_{it+1}^K + q_{it}^B U_{it+1}^B - B_{it+1} = E_t \left[ M_{t+1} \left[ \begin{array}{c} D_{it+1} + q_{it+1}^P K_{it+2}^P \\ + q_{it+1}^L L_{it+2} + q_{it+1}^K U_{it+2}^K + q_{it+1}^B U_{it+2}^B - B_{it+2}. \end{array} \right] \right]$$

Recursively applying the above the equation to future periods,

$$\begin{aligned}
& q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K U_{it+1}^K + q_{it}^B U_{it+1}^B - B_{it+1} \\
= & E_t [M_{t+1} D_{it+1} + M_{t+2} D_{it+2} + M_{t+2} [q_{it+2}^P K_{it+3}^P + q_{it+2}^L L_{it+3} + q_{it+2}^K U_{it+3}^K + q_{it+2}^B U_{it+3}^B - B_{it+3}]] \\
= & \dots \\
= & \sum_{\Delta t=1}^{\infty} M_{t+\Delta t} D_{it+\Delta t} + \lim_{\Delta t \rightarrow \infty} E_t [M_{t+1} [q_{it+\Delta t}^P K_{it+\Delta t}^P + q_{it+\Delta t}^L L_{it+\Delta t} + q_{it+\Delta t}^K U_{it+\Delta t}^K + q_{it+\Delta t}^B U_{it+\Delta t}^B - B_{it+\Delta t}].]
\end{aligned}$$

Assuming the transversality condition holds then,

$$q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K U_{it+1}^K + q_{it}^B U_{it+1}^B = V_{it} - D_{it} + B_{it+1} = P_{it} + B_{it+1}.$$

## B Aggregation Bias in BXZ/LWZ and Alternative Estimation Procedures

In this appendix, we use artificial data to investigate the ability of the different estimation approaches to recover the underlying firm-level structural parameters. We document that the parameter estimates using the aggregation procedure in Liu, Whited and Zhang (2009) (LWZ) do not have a structural interpretation. In addition, we verify that the alternative portfolio-level estimation methods proposed in the main text allow us to recover the firm-level structural parameters.

For simplicity, we consider the one-capital input model. To proceed, we generate data from a model economy in which the assumptions of the baseline investment model hold (and hence the firm-level observed and predicted (model-implied) valuation ratios are equal). But instead of simulating data from a model economy, we use the real data as follows. We construct the capital stock process for each firm by using the law of motion:

$$K_{it} = (1 - \delta)K_{it-1} + I_{it}. \quad (\text{B.1})$$

We use the firm-level physical capital investment data for  $I_{it}$  and the initial capital stock of the firm to be  $K_0$  and assume a depreciation of 10%. To generate price data in this economy, we use the valuation equation implied by the neoclassical model, that is:

$$VR_{it} = 1 + (1 - \tau_t)\theta^2 \frac{I_t}{K_{t-1}}, \quad (\text{B.2})$$

where  $VR_{it} \equiv \frac{P_{it}}{K_{it}}$  in which  $P_{it}$  is the market value of equity. Thus, by construction, the observed and the model-implied valuation ratio are equal.

The econometric exercise of interest here is to investigate the extent to which the different estimation approaches allow us to recover the structural parameters, which in our case is the parameter  $\theta$  (we ignore the estimation of the curvature parameter here for simplicity). To make the results more general, we consider three values of the slope adjustment cost parameters  $\theta = 10$ ,

20, or 40. The curvature is fixed at 2 (quadratic). Given these parameters, we can generate a time series of valuation ratios in the model using equation (B.2).

To examine the role of the impact of portfolio-level aggregation of the firm characteristics using the LWZ procedure, we first create 10 and 50 portfolios sorted on the firm-level lagged valuation ratio (VR) and investment-rate (IK). As in LWZ, we construct the portfolio-level counterpart of the valuation ratio as follows. For each portfolio  $j = 1, \dots, 10, \text{ or } 50$ , and in each period, we have:

$$VR_{jt} = \frac{\sum_i^N P_{it}}{\sum_i^N K_{it}}, \quad i \in \text{Portfolio } j \quad (\text{B.3})$$

$$I_{jt}/K_{jt-1} = \frac{\sum_i^N I_{it}}{\sum_i^N K_{it-1}}. \quad (\text{B.4})$$

To estimate the model parameters we construct the model-implied predicted valuation ratio  $\widehat{VR}_{jt}$  as:

$$\widehat{VR}_{jt} \equiv 1 + (1 - \tau_t)\hat{\theta}^2 \frac{I_t}{K_{t-1}}$$

which uses the portfolio-level investment rate computed as in equation (B.4). Following LWZ, we estimate the model parameters ( $\theta$ ) by the Generalized Method of Moments (GMM) using the moment condition:

$$E \left[ VR_{jt} - \widehat{VR}_{jt} \right] = 0, \quad j = 1, \dots, 10 \text{ or } 50. \quad (\text{B.5})$$

We use the identity matrix as the weighting matrix. We label this method as GMM-XS. For comparison with the estimation approach used here that matches the time series data (and to establish that the conclusions here do not depend on the estimation approach used), we also estimate the parameters by minimizing the sum of squared residuals. That is, let

$$\varepsilon_{jt} = VR_{jt} - \widehat{VR}_{jt}.$$

We then estimate the model parameters using the first order conditions from the minimization of  $\sum_{t=1}^T \sum_{j=1}^N \varepsilon_{jt}^2$ . We denote this method as NLLS -TS. For each estimation method, we report the parameter estimate of the slope coefficient  $\theta$  (reported as  $\hat{\theta}$ ) for the three cases  $\theta = 10, 20,$

or 40, together with the estimation bias, computed as the percentage deviation of the estimated parameter value relative to the true parameter value (bias =  $\frac{\hat{\theta} - \theta}{\theta}$ ).

[Insert Table 7 here]

Table 7, rows LWZ, report the estimation results using the LWZ aggregation method. Rows XSMED report the results with our aggregation using the cross-sectional median, and rows XSEW using the equally weighted mean. Panel A reports the results using the 10 valuation ratio (VR) portfolios, and Panel B reports the results using 50. Panel C reports the results using the 10 investment rate (IK) portfolios, and Panel D reports the results using 50. The columns on the right report the results using the GMM-XS estimation approach (that is, matching the cross section average of each series), while the columns on the left report the results using NLLS-TS estimation approach (that is, matching the time series of the series).

Table 7 reveals that, across all cases, the parameter estimates using the LWZ aggregation procedure differ from the true firm-level structural parameters, and hence do not have a structural interpretation. In all cases considered here, the bias in the estimation ranges from -73.84% to 8.52%, and is never zero. Also, the parameter estimates vary significantly across the set of test assets used for the estimation (IK or VR portfolios), across the number of portfolios (10 vs 50) and across the estimation procedures (GMM-XS vs NLLS-TS), which should not occur in large samples if the estimation procedure is consistent, in which case the procedure should recover the true underlying parameter values. Indeed, the variation of the parameter estimates across test assets helps us understand why the parameter estimates in LWZ vary significantly across different test assets used in the estimation. The bias occurs here because of aggregation issues in the procedure. The nonlinearities in the valuation ratio mean that the true portfolio-level valuation ratio is different from the portfolio-level valuation ratio obtained by first aggregating each portfolio-level characteristics (investment rate, etc.) separately, to construct the portfolio-level valuation ratio counterparts. A larger number of portfolios and a estimation procedure that takes the time-

series into account minimizes the bias because it decreases aggregation.

Turning to the analysis of the alternative estimation procedures discussed in the main text, namely the cross sectional mean aggregation (XSMED) and the equal-weighted aggregation (XSEW), Table 7 shows that these methods avoid the aggregation issues in LWZ. In particular, the results in Table 7 show that the three alternative aggregation procedures are unbiased, thus allowing us to recover the true underlying firm-level structural parameters.

Naturally, with measurement error, the analysis becomes significantly more complicated. Since measurement error in firm-level data is not directly observed, different assumptions about the nature of the error may lead to different results. This does not invalidate the previous analysis. The analysis here shows that even without measurement error, the aggregation procedure in LWZ contaminates the parameter estimates, which in turn invalidates the interpretation of the parameter estimates as firm-level structural parameters. While it is theoretically possible that measurement error in the data might lead an inconsistent estimation method to recover the true parameter values in the data, this is unlikely to be case here, especially when a large set of moments and a large set of test assets is used in the estimation.

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## Figures and Tables

**Table 1 : Summary Statistics**

Panel A reports the time-series average of the cross-sectional median, and the standard-deviation of selected characteristics of the firm level data.  $VR_{it}$  is the firm's valuation ratio.  $I_{it}^P/K_{it}^P$  is the investment rate in physical capital,  $H_{it}/L_{it}$  is the investment rate in labor stock (hiring rate),  $I_{it}^K/U_{it}^K$  is the investment rate in knowledge capital and  $I_{it}^B/U_{it}^B$  is the investment rate in brand capital. We also present the stock of each input (physical capital, labor, knowledge capital and brand capital) relative to the sum of the three capital inputs. The results are presented for all firms and for each of the three labor skill industries. Panel B shows cross-correlations of the investment/hiring rates for all firms. The sample is annual data from 1975 to 2016.

Panel A: Descriptive Statistics

	Median				S.D.			
	All	Low S.	Med S.	High S.	All	Low S.	Med S.	High S.
Valuation ratios								
$VR_{it}$	1.97	1.51	1.73	2.18	4.27	2.91	3.39	4.73
Investment/hiring rates								
$I_{it}^P/K_{it}^P$	0.22	0.14	0.16	0.29	0.79	0.54	0.54	0.90
$H_{it}/L_{it}$	0.17	0.17	0.15	0.18	0.30	0.28	0.26	0.31
$I_{it}^K/K_{it}^K$	0.26	0.20	0.21	0.31	0.26	0.19	0.19	0.29
$I_{it}^B/K_{it}^B$	0.24	0.23	0.23	0.26	0.27	0.21	0.19	0.30
Scaled capital and labor ratios								
$K_{it}^P/A_{it}$	0.46	0.65	0.60	0.36	0.27	0.24	0.25	0.25
$(W_{it-1}L_{it})/A_{it}$	0.63	0.60	0.67	0.63	1.95	3.36	2.05	1.57
$U_{it}^K/A_{it}$	0.31	0.08	0.16	0.16	0.27	0.15	0.20	0.26
$U_{it}^B/A_{it}$	0.11	0.20	0.14	0.10	0.17	0.19	0.19	0.15

Panel B: Correlations all firms

	$H_{it}/L_{it}$	$I_{it}^K/K_{it}^K$	$I_{it}^B/K_{it}^B$
$I_{it}^P/K_{it}^P$	0.49	0.39	0.39
$H_{it}/L_{it}$	—	0.36	0.38
$I_{it}^K/K_{it}^K$	—	—	0.56

**Table 2 : Firm Value Decomposition Based on Book Values**

This table reports the fraction of firm value that is attributed to each input ( $\mu$ ) based on its book value. This decomposition is done by setting all the adjustment costs to zero and evaluating at the median value of the ratio of the capital inputs for the period between 1975 to 2016. The results are reported for all firms, low-, mid-, and high-skill industries.

	All	Low S.	Mid S.	High S.
$\bar{\mu}^P$ : Physical capital	62.41	78.16	75.31	49.73
$\bar{\mu}^L$ : Labor	0.00	0.00	0.00	0.00
$\bar{\mu}^K$ : Knowledge capital	27.55	5.97	13.21	41.44
$\bar{\mu}^B$ : Brand capital	10.04	15.87	11.48	8.83

**Table 3 : Estimation Across Labor Skill Industries**

This table reports estimation results, measures of fit and the implied firm value decomposition. The columns show the values for different model specifications for all firms, low-, mean- and high-labor skill industries. The estimation uses five portfolios based on the lagged value ratio. The estimation is done using the cross-sectional median aggregation method and NLLS methodology. Panel A reports the estimation results.  $\theta_P$ ,  $\theta_L$ ,  $\theta_K$  and  $\theta_B$  are respectively, the physical capital, labor, knowledge capital and brand capital slope adjustment cost parameters.  $\nu_P$ ,  $\nu_L$ ,  $\nu_K$  and  $\nu_B$  are, respectively, the physical capital, labor, knowledge capital and brand capital curvature adjustment cost parameters. s.e. stands for bootstrapped standard errors. Panel B reports measures of fit and value decomposition. m.a.e. is the mean absolute error of the valuation error, and m.a.e./|VR| is the mean absolute valuation error scaled by the absolute value of the ratio. The table reports the median fraction of the value that is attributed to each input ( $\mu$ ). CX/Y is the ratio (in percent) of the implied input adjustment costs-to-sales ratio. The sample is annual data from 1975 to 2016.

		Panel A: Parameter estimates											
Column number:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	
Skill:	All	L	M	H	L	H	L	H	L	H	L	H	
Slope													
$\theta_P$	2.90	7.16	3.38	3.44	6.88	6.46	3.53	3.48	7.09	6.04	7.45	5.47	
s.e.	[0.58]	[1.40]	[1.12]	[0.82]	[1.99]	[0.68]	[1.54]	[0.57]	[1.56]	[0.85]	[1.86]	[0.70]	
$\theta_L$	5.74	3.75	4.14	5.75			4.42	5.18					
s.e.	[0.56]	[2.29]	[0.77]	[0.58]			[2.68]	[0.56]					
$\theta_K$	2.99	0.66	6.94	3.94					1.31	5.39			
s.e.	[0.62]	[3.15]	[1.38]	[0.60]					[2.98]	[0.65]			
$\theta_B$	2.21	7.93	3.67	2.20							7.56	3.52	
s.e.	[0.73]	[2.32]	[1.02]	[0.76]							[2.13]	[1.16]	
Curvature													
$\nu_P$	2.43	1.50	2.45	1.72	1.82	2.04	2.35	2.21	1.83	1.74	1.82	2.02	
s.e.	[0.36]	[1.07]	[0.58]	[0.34]	[1.64]	[0.17]	[1.43]	[0.31]	[1.13]	[0.19]	[1.41]	[0.17]	
$\nu_L$	1.68	1.37	1.13	1.78			1.38	1.96					
s.e.	[0.24]	[1.37]	[0.36]	[0.27]			[0.58]	[0.30]					
$\nu_K$	1.27	3.89	1.49	1.77					0.22	1.98			
s.e.	[0.29]	[1.11]	[0.39]	[0.26]					[1.34]	[0.20]			
$\nu_B$	2.35	1.15	2.27	1.37							0.79	3.43	
s.e.	[0.45]	[1.50]	[1.18]	[0.56]							[0.95]	[0.66]	
Intercept	-1.77	-3.97	-2.36	-2.60	-0.82	-0.13	-0.63	-0.48	-1.44	-2.70	-2.67	-0.01	
s.e.	[0.35]	[1.01]	[0.53]	[0.45]	[0.84]	[0.26]	[1.24]	[0.26]	[0.74]	[0.58]	[0.80]	[0.27]	

Table 3 : Estimation Across Labor Skill Industries (cont.)

		Panel B: Measures of fit, implied firm value decomposition, and adjustment costs											
Column number:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	
Skill:	All	L	M	H	L	H	L	H	L	H	L	H	
Measures of fit													
XS- $R^2$	0.99	0.96	0.97	0.98	0.95	0.95	0.95	0.98	0.96	0.97	0.97	0.94	
TS- $R^2$	0.80	0.65	0.75	0.80	0.53	0.62	0.57	0.75	0.57	0.76	0.61	0.60	
m.a.e.	0.75	0.87	0.73	0.84	0.98	1.03	0.93	0.87	0.94	0.84	0.86	1.08	
m.a.e./ $VR$	0.26	0.36	0.29	0.27	0.40	0.33	0.38	0.28	0.38	0.27	0.35	0.34	
Firm value decomposition (in %)													
$\bar{\mu}^P$ : Physical capital	25.07	56.95	29.34	22.67	100.00	100.00	48.04	41.28	92.34	44.78	72.38	89.27	
$\bar{\mu}^L$ : Labor	49.05	18.20	33.84	40.83	-	-	51.96	58.72	-	-	-	-	
$\bar{\mu}^K$ : Knowledge capital	21.85	0.89	27.33	33.00	-	-	-	-	7.66	55.22	-	-	
$\bar{\mu}^B$ : Brand capital	4.03	23.97	9.50	3.51	-	-	-	-	-	-	27.62	10.73	
Adjustment costs (in %)													
$CP/Y$ : Physical capital	3.81	18.35	2.83	11.15	15.27	35.71	2.76	9.67	16.09	29.45	17.66	25.45	
$CL/Y$ : Labor	22.30	10.31	16.48	26.93	-	-	12.75	20.68	-	-	-	-	
$CK/Y$ : Knowledge capital	9.05	0.00	8.37	20.95	-	-	-	-	10.57	36.26	-	-	
$CB/Y$ : Brand capital	0.52	13.63	1.62	1.62	-	-	-	-	-	-	15.31	1.07	

**Table 4 : Decomposing Firm Value Across Decades**

This table reports the average value of the median fraction of the value that is attributed to each input across different decades. The calculations are done using the estimates of Table ?? column (1) for all firms and 3 column (1)-(3) for each labor skill. The sample is annual data from 1975 to 2016.

	1970s	1980s	1990s	2000s	2010s
All Firms					
$\bar{\mu}^P$ : Physical capital	30.86	31.95	23.06	20.34	20.72
$\bar{\mu}^L$ : Labor	55.15	46.30	51.42	47.21	47.85
$\bar{\mu}^K$ : Knowledge capital	9.34	16.54	21.26	29.24	28.67
$\bar{\mu}^B$ : Brand capital	4.65	5.21	4.27	3.21	2.75
Low Skill					
$\bar{\mu}^P$ : Physical capital	62.52	61.00	55.19	53.74	54.29
$\bar{\mu}^L$ : Labor	19.26	17.62	19.44	18.63	15.87
$\bar{\mu}^K$ : Knowledge capital	0.62	0.92	1.02	0.99	0.69
$\bar{\mu}^B$ : Brand capital	17.61	20.47	24.36	26.64	29.15
Mid Skill					
$\bar{\mu}^P$ : Physical capital	32.47	34.60	28.82	25.88	25.26
$\bar{\mu}^L$ : Labor	40.61	29.92	36.73	32.47	32.45
$\bar{\mu}^K$ : Knowledge capital	16.39	24.01	24.60	33.34	35.18
$\bar{\mu}^B$ : Brand capital	10.54	11.48	9.85	8.31	7.11
High Skill					
$\bar{\mu}^P$ : Physical capital	32.08	31.17	20.53	16.09	16.24
$\bar{\mu}^L$ : Labor	47.61	37.13	42.00	39.51	41.46
$\bar{\mu}^K$ : Knowledge capital	15.97	27.26	33.65	41.67	40.05
$\bar{\mu}^B$ : Brand capital	4.33	4.44	3.81	2.73	2.25

**Table 5 : Business Cycle Properties of the Value and Shares of the Capital and Labor Inputs**

This table reports the cyclicity of the value ratio and the cyclicity and covariance of each component of the decomposition. The cyclical component is calculated using an HP-filter, with a smoothing factor of 100, on the log of each portfolio time series and an HP filter on the log of the aggregate sales time series. The correlations are also calculated using the HP filtered series. Panel A displays the cyclicity of the value ratio (VR), the shares ( $\mu$ ) and value components (V) and the standard deviation of each series for all firms and high and low skill. Panel B displays the cross input correlations of the shares and value components. The sample is annual data from 1975 to 2016.

Industry:	Cyclicity (Cov. with $Y^{\text{agg}}$ )*100					S.D.			
	All	Low S.	Med. S.	High S.	All	Low S.	Med. S.	High S.	
$VR$ : Valuation ratio	0.28	0.07	0.15	0.33	0.16	0.08	0.14	0.15	
	Capital/labor shares (HP cycle)								
$\bar{\mu}^P$ : Physical capital	-0.04	0.03	0.10	-0.04	0.13	0.05	0.12	0.13	
$\bar{\mu}^L$ : Labor	0.16	-0.02	-0.28	0.26	0.13	0.08	1.15	0.16	
$\bar{\mu}^K$ : Knowledge capital	-0.35	-0.12	-0.16	-0.33	0.17	0.13	0.15	0.14	
$\bar{\mu}^B$ : Brand capital	-0.20	-0.05	-0.06	-0.33	0.13	0.11	0.12	0.13	
	Capital/labor values (HP cycle)								
$V^P$ : Physical capital	0.25	0.10	0.26	0.29	0.09	0.09	0.10	0.09	
$V^L$ : Labor	0.42	0.03	0.06	0.55	0.26	0.13	0.64	0.28	
$V^K$ : Knowledge capital	-0.07	-0.05	0.00	0.00	0.05	0.10	0.05	0.05	
$V^B$ : Brand capital	0.09	0.02	0.09	0.00	0.05	0.09	0.07	0.05	

**Table 6 : Robustness Checks**

This table reports the estimation results and measures of fit and value decomposition. Panel A reports the estimation results.  $\theta_P$ ,  $\theta_L$ ,  $\theta_K$  and  $\theta_B$  are respectively, the physical capital, labor, knowledge capital and brand capital slope adjustment cost parameters.  $\nu_P$ ,  $\nu_L$ ,  $\nu_K$  and  $\nu_B$  are, respectively, the physical capital, labor, knowledge capital and brand capital curvature adjustment cost parameters. s.e. stands for bootstrapped standard errors. Panel B reports measures of fit and value decomposition. m.a.e. is the mean absolute error of the valuation error, and m.a.e./|VR| is the mean absolute valuation error scaled by the absolute value of the ratio. The columns show values for different aggregation methods and for different number of portfolios based on the lagged value ratio. XSMED is the cross-sectional median aggregation method and XSEW is the cross-sectional equal-weighted aggregation method.  $VR_{75-25}$  minimizes the difference between the estimated error of quantile 75 and the quantile 25 while  $VR_{25,50,75}$  minimizes the estimation error of quantiles 25, 50 and 75. Finally, FL performs the estimation using firm level data. Panel B reports measures of fit and value decomposition. m.a.e. is the mean absolute error of the valuation error, and m.a.e./|VR| is the mean absolute valuation error scaled by the absolute value of the ratio. The table reports the median fraction of the value that is attributed to each input ( $\mu$ ). CX/Y is the ratio (in percent) of the implied input adjustment costs-to-sales ratio. The sample is annual data from 1975 to 2016.

	Panel A: Parameter estimates							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Column number:	5	2	10	20	5	5	5	—
Number of portfolios:	5	2	10	20	5	5	5	—
Slope								
$\theta_P$	2.90 [0.58]	3.24 [0.25]	3.97 [0.94]	4.17 [1.02]	4.11 [0.57]	0.58 [0.49]	3.43 [0.59]	3.09 [0.27]
s.e.	5.74	3.07	5.32	5.04	4.93	3.65	3.90	2.06
$\theta_L$	[0.56]	[0.43]	[0.68]	[0.68]	[0.45]	[0.39]	[0.39]	[0.39]
s.e.	2.99	3.84	4.51	4.82	3.75	2.21	4.39	5.87
$\theta_K$	[0.62]	[0.36]	[0.60]	[0.58]	[0.43]	[0.57]	[0.38]	[0.59]
s.e.	2.21	2.16	2.17	1.46	0.00	2.04	1.97	3.73
$\theta_B$	[0.73]	[0.45]	[1.16]	[1.16]	[1.68]	[0.60]	[0.72]	[0.67]
s.e.	2.43	2.97	1.95	1.60	1.84	2.18	1.69	1.63
Curvature	[0.36]	[0.33]	[0.34]	[0.45]	[0.13]	[0.67]	[0.22]	[0.04]
$\nu_P$	1.68	1.29	1.62	2.36	1.72	2.48	2.25	1.65
s.e.	[0.24]	[0.24]	[0.34]	[0.63]	[0.16]	[0.40]	[0.33]	[0.47]
$\nu_K$	1.27	1.71	1.72	1.45	1.63	2.37	1.75	1.00
s.e.	[0.29]	[0.19]	[0.22]	[0.23]	[0.15]	[0.35]	[0.20]	[0.12]
$\nu_B$	2.35	2.56	1.47	1.08	1.98	2.01	1.55	2.00
s.e.	[0.45]	[0.36]	[0.50]	[0.44]	[0.32]	[0.64]	[0.33]	[0.51]
Intercept	-1.77	-1.31	-2.61	-2.42	-2.31	0.50	-1.52	-0.56
s.e.	[0.35]	[0.32]	[0.52]	[0.51]	[0.26]	[0.00]	[0.31]	[6.19]

Table 6 : Robustness Checks (cont.)

Panel B: Measures of fit, implied firm value decomposition, and adjustment costs							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	XSMED	XSEW	$VR_{75-25}$	$VR_{25,50,75}$	FL		
Column number:	5	2	10	20	5	5	5
Number of portfolios:							
Measures of fit							
XS- $R^2$	0.99	0.99	0.98	0.98	0.98	0.67	0.92
TS- $R^2$	0.80	0.78	0.80	0.77	0.83	0.58	0.79
m.a.e.	0.75	0.57	0.85	0.97	0.85	1.12	0.94
m.a.e./ VR	0.26	0.22	0.28	0.31	0.25	0.39	0.31
Firm value decomposition (in %)							
$\bar{\mu}^P$ : Physical capital	25.07	33.31	30.44	34.06	36.16	29.19	35.52
$\bar{\mu}^L$ : Labor	49.05	28.28	36.79	30.94	36.35	34.86	23.01
$\bar{\mu}^K$ : Knowledge capital	21.85	33.75	28.58	31.17	25.69	27.19	36.49
$\bar{\mu}^B$ : Brand capital	4.03	4.67	4.18	3.83	1.80	8.75	4.97
Adjustment costs (in %)							
CP/Y : Physical capital	3.81	3.90	9.81	12.53	10.61	0.13	8.58
CL/Y : Labor	22.30	12.32	20.29	12.94	16.67	5.56	7.61
CK/Y : Knowledge capital	9.05	9.41	12.33	15.28	9.63	1.84	11.62
CB/Y : Brand capital	0.52	0.39	1.39	1.56	0.00	0.64	1.07
							2.15
							7.34
							4.12
							24.25
							2.15

**Table 7 : Comparison of Estimation Methods: the Impact of Portfolio-Level Aggregation**

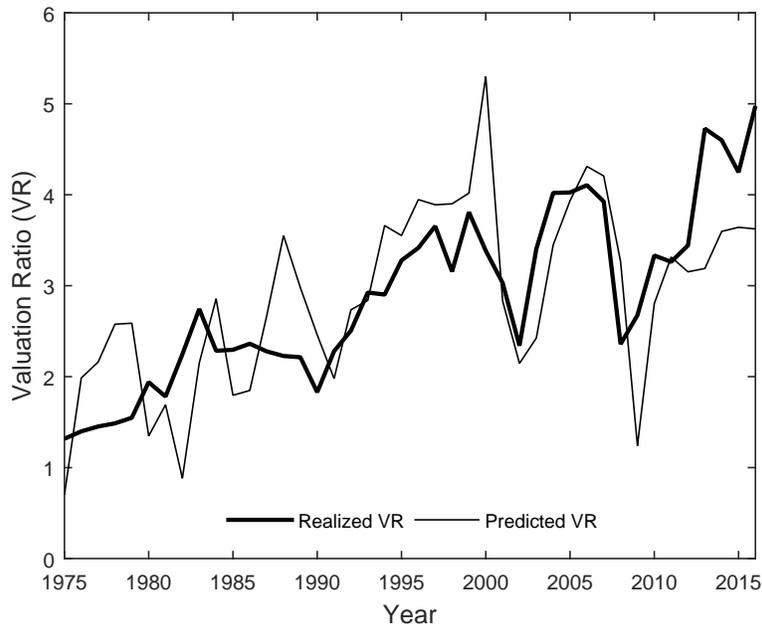
This table reports the estimates of the model parameters across different portfolio-level aggregation methods for the physical capital only model with curvature equal to 2, the slope is represented by beta as we describe in Section B. We consider three values of true model parameters at the firm level:  $\theta = 10$ ,  $\theta = 20$ , or  $\theta = 40$ . For each method,  $\hat{\theta}$  is the estimated parameter, and bias is the percentage deviation of the estimated parameter value relative to the true parameter value (bias =  $\frac{\hat{\theta} - \theta}{\theta}$ ). In LWZ the data is aggregated by first aggregating the firm characteristics to obtain the portfolio-level predicted valuation ratio as described in Section B. XSMED is the cross-sectional median aggregation method in which we compute the portfolio-level observed and predicted cross sectional median of the valuation ratio across all the firms in the portfolios in each year; XSEW is the equal-weighted cross sectional mean aggregation method in which we compute the portfolio-level observed and predicted cross sectional valuation ratio across all the firms in the portfolios in each year. In Panel A(B), the test assets are 10(50) value ratio portfolios, and in Panel C(D) the test assets are 10(50) investment rate portfolios. Two estimation methods are used. In NLLS-TS the parameters are obtained by minimizing the sum of squared portfolio-level residual (the difference between observed and model-implied valuation ratio) at the portfolio-level. In GMM-XS the parameters are obtained by matching the average observed and predicted valuation ratio of each portfolio (as in LWZ).

True Value:	NLLS-TS						GMM-XS					
	$\theta = 10$		$\theta = 20$		$\theta = 40$		$\theta = 10$		$\theta = 20$		$\theta = 40$	
Estimate:	$\hat{\theta}$	Bias (%)										
Panel A: Estimation across 10 VR Portfolios												
LWZ/BXZ	8.30	-17.00	16.64	-16.80	33.30	-16.75	7.73	-22.70	15.48	-22.60	30.96	-22.60
XSMED	10.00	0.00	20.00	0.00	40.00	0.00	10.00	0.00	20.00	0.00	40.00	0.00
XSEW	10.00	0.00	20.00	0.00	40.00	0.00	10.00	0.00	20.00	0.00	40.00	0.00
Panel B: Estimation across 50 VR Portfolios												
LWZ/BXZ	8.31	-16.90	16.66	-16.70	33.33	-16.68	7.74	-22.60	15.49	-22.50	31.00	-22.50
XSMED	10.00	0.00	20.00	0.00	40.00	0.00	10.00	0.00	20.00	0.00	40.00	0.00
XSEW	10.00	0.00	20.00	0.00	40.00	0.00	10.00	0.00	20.00	0.00	40.00	0.00
Panel C: Estimation across 10 IK Portfolios												
LWZ/BXZ	8.72	-12.80	17.45	-12.75	34.91	-12.73	8.36	-16.40	16.72	-16.40	33.44	-16.40
XSMED	10.00	0.00	20.00	0.00	40.00	0.00	10.00	0.00	20.00	0.00	40.00	0.00
XSEW	10.00	0.00	20.00	0.00	40.00	0.00	10.00	0.00	20.00	0.00	40.00	0.00
Panel D: Estimation across 50 IK Portfolios												
LWZ/BXZ	9.86	-1.40	19.73	-1.35	39.47	-1.32	9.73	-2.70	19.45	-2.75	38.91	-2.73
XSMED	10.00	0.00	20.00	0.00	40.00	0.00	10.00	0.00	20.00	0.00	40.00	0.00
XSEW	10.00	0.00	20.00	0.00	40.00	0.00	10.00	0.00	20.00	0.00	40.00	0.00

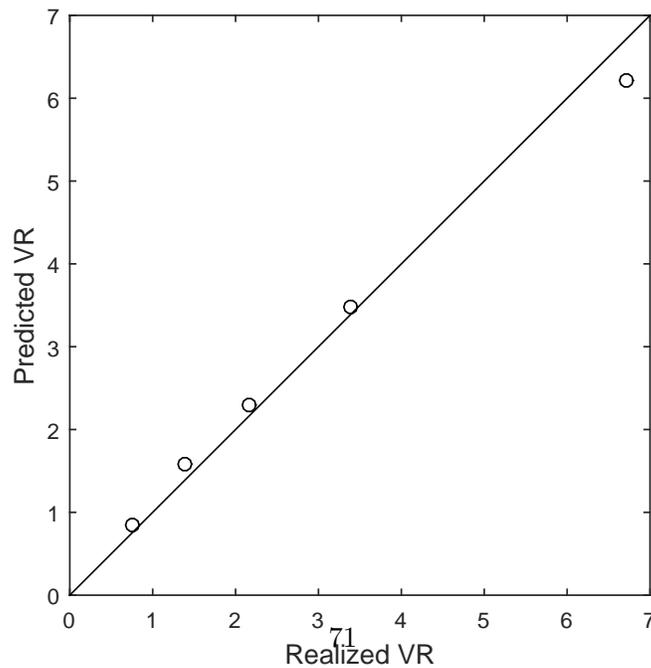
**Figure 1 : Time-Series and Cross-Sectional Fit of the Baseline Model**

Panel A plots the predicted versus realized time series of the average (across portfolios) of the valuation ratio from the estimation of the investment-based model using the cross sectional median (XSMED) estimation method and five valuation ratio portfolios as test assets. Panel B plots, for each portfolio, the time-series average of the predicted and realized cross sectional median valuation ratios. The sample is annual data from 1975 to 2016.

Panel A: Time-series fit



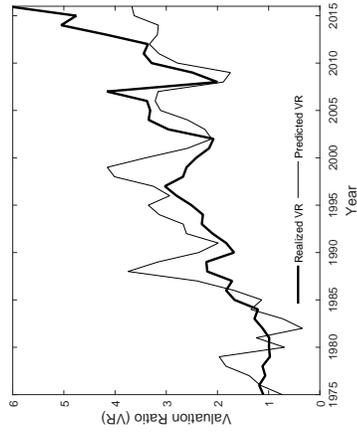
Panel B: Cross-sectional fit



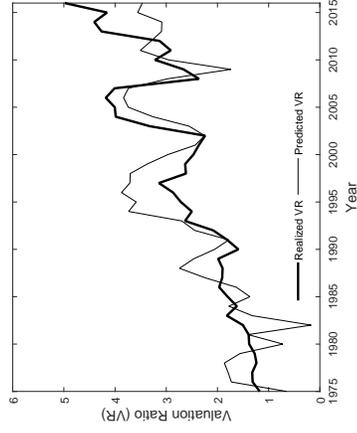
**Figure 2 : Time-Series and Cross-Sectional Fit Across Industries**

Panel A (C,E) plots the predicted versus realized time series of the average of the valuation ratio for low- (mid-, high-) skill industries from the estimation of the investment-based model using the cross sectional median (XSMED) estimation method and five valuation ratio portfolios as test assets. Panel B (D, F), plots the average of the across time for low (mid, high) skill industries for the predicted and realized cross sectional median valuation ratio. The sample is annual data from 1975 to 2016.

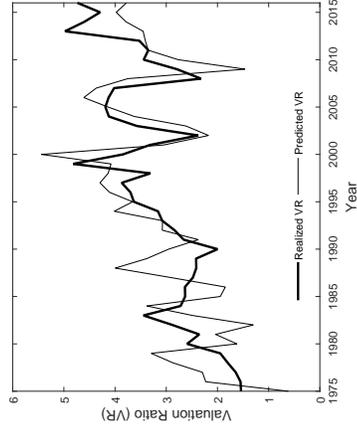
**Panel A: TS Fit, Low Skill**



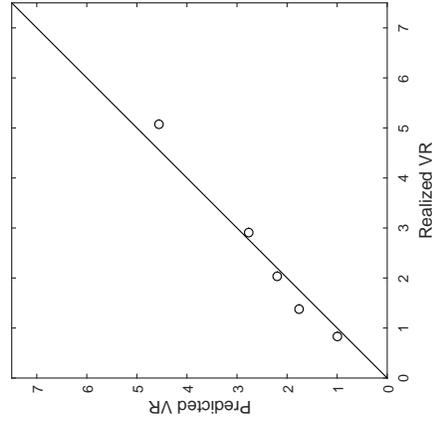
**Panel C: TS Fit, Mid Skill**



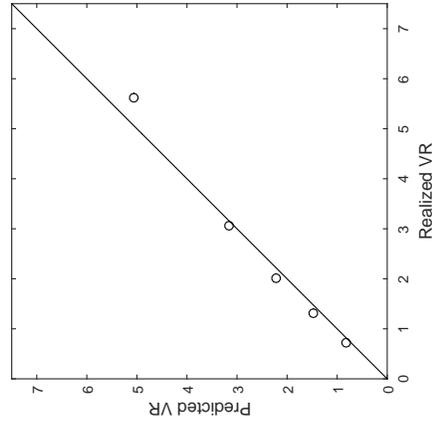
**Panel E: TS Fit, High Skill**



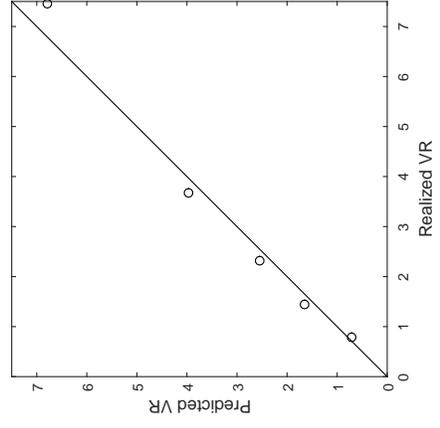
**Panel B: XS Fit, Low Skill**



**Panel D: XS Fit, Mid Skill**



**Panel F: XS Fit, High Skill**



**Figure 3 : Contribution of Each Input to Firm’s Market Value Over Time**

This figure plots the time series of the median contribution of each input for firms’ market value (shares) implied by the estimation of the neoclassical investment model using the portfolio-level cross sectional median (XSMED) estimation method, and five portfolios as test assets.  $\mu_P$  is the share of physical capital,  $\mu_L$  is the share of labor,  $\mu_K$  is the share of knowledge capital and  $\mu_B$  is the share of brand capital. Panel A shows the results across all firms, Panel B shows the results across low skill industries, Panel C shows the results across mid skill industries, and Panel D across high skill industries. The sample is annual data from 1975 to 2016.

