

# A Multivariate Stochastic Unit Root Model with an Application to Derivative Pricing\*

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## Abstract

This paper extends recent findings of Lieberman and Phillips (2014) on stochastic unit root (STUR) models to a multivariate case including a comprehensive asymptotic theory for estimation of the model's parameters. The extensions are useful because they lead to a generalization of the Black-Scholes formula for derivative pricing. In place of the standard assumption that the price process follows a geometric Brownian motion, we derive a new form of the Black-Scholes equation that allows for a multivariate time varying coefficient element in the price equation. The corresponding formula for the value of a European-type call option is obtained and shown to extend the existing option price formula in a manner that embodies the effect of a stochastic departure from a unit root. An empirical application reveals that the new model substantially reduces the average percentage pricing error of the Black-Scholes and Heston's (1993) stochastic volatility pricing schemes in most moneyness-maturity categories considered.

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