Efficient Taxation of Labour Income under the Threat of Conflict

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Working Paper No. 990 August 2025 ISSN 1473-0278

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August 18, 2025

Abstract

I take a simple model of affine taxation of labour income and append to it the possibility that the chosen tax schedule triggers conflict in society. I demonstrate theoretically that, under certain conditions, the set of efficient tax schedules is a proper subset of the set of efficient tax schedules in a standard model without the possibility of conflict. Then, calibrating the model to the United States, I show numerically that the former set is much smaller than the latter.

Keywords: labour-income tax; redistribution; conflict

JEL classification: D74, H20, H21

Declarations of interest: none

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1 Introduction

Characterising the set of efficient tax schedules over labour income is an important topic in public economics.[1] However, in examining the efficiency of a tax schedule, the papers in this literature do not allow for the possibility that it may trigger social conflict (e.g., in the form of lobbying, election campaigns, time spent on political discourse, and even violent clashes).

In the current paper, I take a simple model with affine taxation of labour income and append to it the threat of conflict (TC). In particular, (i) society makes an initial choice of a tax rate,[2] (ii) each productivity type can challenge this tax rate, (iii) if no type challenges, this tax rate is implemented, (iv) a challenge by any type triggers conflict in which types expend resources on conflict, (v) types' expenditures on conflict determine their probabilities of "winning" the conflict, and (vi) the winning type gets to set the final tax rate.

The central question I address is how the set of efficient tax rates under TC (i.e., the set of initially chosen tax rates in (i) that are efficient given (ii)-(vi)) compares to the set of efficient tax rates in a standard model without TC. I demonstrate theoretically that, under certain conditions, the former set is a proper subset of the latter set. Then, calibrating the model to the United States, I show numerically that the difference between the two sets is quantitively very large.

[1] In a setting in which the government can use only affine tax schedules, the revenue-maximising tax rate is strictly below 1 (as illustrated by the famous Laffer curve), and tax rates above it are inefficient. In the context of nonlinear taxation, a seminal early contribution is Werning (2007), and a more recent contribution is Bierbrauer et al. (2023).

[2] The tax rate pins down the intercept of the tax schedule (and, hence, the whole tax schedule) through the government budget constraint.

2 The Model

2.1 Types

There are types $i = 1, \ldots, I$, where $I \geq 2$. Type i has hourly wage w_i, where $0 \leq w_1 < \ldots < w_I$. The proportion of individuals of type i is $p_i > 0$.

2.2 Preferences

Each type has a von Neumann-Morgenstern utility function over consumption and labour (measured in hours) that is of the form $c - \beta \frac{\epsilon}{1+\epsilon} l^{\frac{1+\epsilon}{\epsilon}}$, where $\beta > 0$ and $\epsilon > 0$. Thus, preferences exhibit quasilinearity in consumption, constant (Hicksian and Marshallian) elasticity of labour supply (which is equal to ϵ), and risk neutrality with respect to lotteries over consumption.

2.3 Initial Choice of Tax Schedule

Society makes an initial choice of a tax schedule over labour income. I assume this choice is restricted to affine tax schedules, i.e., to tax schedules of the form $\tau(y) = -T + ty$, where y is pre-tax labour income, $T \geq 0$ is the guaranteed post-tax income of somebody with zero pre-tax labour income, and $t \leq 1$ is the tax rate.[3] The tax schedule has to satisfy the government budget constraint

$$t \sum_{i=1}^{I} p_i w_i l_i(t) = T + G,$$

where $G \geq 0$ is the exogenously given government expenditure per-capita and $l_i(t) = \frac{(1-t)^\epsilon w_i^\epsilon}{\beta^\epsilon}$ is the labour supplied by type i.[4] I assume that $t \sum_{i=1}^{I} p_i w_i l_i(t) \geq G$ at the

[3] Tax rates above 1 lead to the exact same outcome as $t = 1$ (namely, $y = 0$ for all types).

[4] Type i facing tax rate t chooses l by solving $\max_{l \geq 0} (1-t) w_i l + T - \beta \frac{\epsilon}{1+\epsilon} l^{\frac{1+\epsilon}{\epsilon}}$. It is straightforward to establish that the unique solution to this problem is $l = \frac{(1-t)^\epsilon w_i^\epsilon}{\beta^\epsilon}$.

tax revenue-maximising tax rate, $t = \frac{1}{1+\epsilon}$.

The following notation will be useful. Let $T(t)$ denote the level of T that is determined by t through the government budget constraint. (From here on, I will identify tax rate t with the tax schedule $\tau(y) = -T(t) + ty$.) Define type i's indirect utility under tax rate t as

$$u_i(t) = (1-t)w_i l_i(t) + T(t) - \beta\frac{\epsilon}{1+\epsilon} l_i(t)^{\frac{1+\epsilon}{\epsilon}}.$$

Next, let $t_{\min}$ ($t_{\max}$, respectively) denote the lowest (highest, respectively) tax rate that covers government expenditures.[5] Finally, let t_i denote type i's optimal tax rate, i.e., the solution to $\max_{t\leq 1, T(t)\geq 0} u_i(t)$.[6]

2.4 Potential Challenge

Given the initially chosen tax rate, each type decides whether to challenge it. If no type challenges, the initial tax rate is adopted. If at least one type challenges, there is conflict (i.e., it takes one to start a fight). Each type i challenges tax rate t if the expected utility type i would obtain in the ensuing game of conflict is strictly higher than $u_i(t)$.

2.5 The Game of Conflict

In case of conflict, each individual of each type i expends $x_i \geq 0$ units of the consumption good on conflict. If $x_1 = \ldots = x_I = 0$, each type i wins with probability

[5] That is, $t_{\min}$ ($t_{\max}$, respectively) is the smaller (larger, respectively) root of $t\sum_{i=1}^{I} p_i w_i l_i(t) = G$.

[6] By examining the derivative of $u_i(\cdot)$, it is straigthforward to establish that $t_i = \begin{cases} \max\left(t_{\min}, \frac{\sum_j p_j w_j^{1+\epsilon} - w_i^{1+\epsilon}}{(1+\epsilon)\sum_j p_j w_j^{1+\epsilon} - w_i^{1+\epsilon}}\right) & \text{if } \sum_j p_j w_j^{1+\epsilon} > w_i^{1+\epsilon} \\ t_{\min} & \text{otherwise} \end{cases}$.

$1/I$; otherwise, type i wins with probability

$$\frac{(p_i x_i)^\alpha}{\sum_{j=1}^{I}(p_j x_j)^\alpha}, \tag{1}$$

where $\alpha > 0$.[7] The type that wins gets to set the final tax rate.

In case of conflict, type i's expected utility given $(x_1, \ldots, x_I)$ is

$$U_i(x_1, \ldots, x_I) = \begin{cases} \frac{1}{I}\sum_{j=1}^{I} u_i(t_j) & \text{if } x_1 = \ldots = x_I = 0 \\ \sum_{j=1}^{I} \frac{(p_j x_j)^\alpha}{\sum_{k=1}^{I}(p_k x_k)^\alpha} u_i(t_j) - x_i & \text{otherwise} \end{cases}.$$

A Nash equilibrium is a tuple $(x_1^*, \ldots, x_I^*)$ such that, for all $i \in \{1, \ldots, I\}$, $x_i^* \in \arg\max_{x_i \geq 0} U_i(x_1^*, \ldots, x_{i-1}^*, x_i, x_{i+1}^*, \ldots, x_I^*)$.

I make two assumptions. First, I assume that $t_i \neq t_j$ for all types i and j such that $i \neq j$. The justification is that, if two distinct types had the same optimal tax rate, they would likely join forces in the conflict so that it would make little sense to treat them as distinct players. This assumption plays a role in the proof of Lemma 1 below and will guide the selection of the number of types, I, in the calibration of the model.

Second, I assume that there exists a unique Nash equilibrium. This assumption underlies the specification in section 2.6 below of how each type evaluates the game of conflict ex ante. The following lemma provides a sufficient condition.[8]

Lemma 1 *If $\alpha < 1$, the game of conflict has a Nash equilibrium. If $\alpha \leq 0.5$, the Nash equilibrium is unique.*[9]

[7] This class of so-called contest success functions has been axiomatised by Skaperdas (1996).

[8] This game of conflict is very similar to the game of conflict analysed in Esteban and Ray (1999). Lemma 1 also lays no claim to novelty as its statement and proof closely follow results in that paper. The game of conflict here and in Esteban and Ray (1999) differs from most of the literature on contests in that a loser cares about who wins. For textbook treatments of the latter literature, see Konrad (2015) and Vojnović (2015).

[9] All proofs are in the appendix.

Note several implicit assumptions in the game of conflict. First, I assume that individuals with different wages form distinct groups for the purposes of conflict. The benefit of this simplification is that it avoids having to model how individuals with different wages form alliances in the conflict. The main cost is that, to avoid $t_i = t_j$ for some distinct types i and j in the calibration exercise below, the empirical distribution of wages will have to be approximated by one with a small number of types.

Second, all individuals of the same type act as one. In particular, there is no issue of free-riding or miscoordination among them.

Third, I allow the x_i's to be arbitrarily large. Thus, I am ignoring any budget constraint each type may be facing.[10]

Fourth, I assume that the initial tax rate plays no role. Thus, I am ruling out that (i) the initial tax rate applies for a period of time until the conflict is resolved and (ii) the initial tax rate establishes a status quo that enjoys some advantage in the conflict. Note that, if (i) or (ii) holds, the initial tax rate becomes more similar to a tax rate chosen without TC (i.e., once and for all as in a standard model). Thus, (i) and (ii) are likely to close any gap between the set of efficient tax rates under TC and the set of efficient tax rates without TC.

Finally, I rule out the possibility to challenge the tax rate set by the winner.

2.6 Expected Utility under TC

Let $(x_1^*, \ldots, x_I^*)$ denote the Nash equilibrium of the game of conflict. Let $Z = \{t \in [t_{\min}, t_{\max}] | u_j(t) \geq U_j(x_1^*, \ldots, x_I^*), \forall j \in \{1, \ldots, I\}\}$, i.e., Z denotes the set of feasible tax rates that do not get challenged by any type.

[10] In the appendix, I provide further comments on this assumption.

The expected utility under TC for each type i given initial tax rate t is

$$u_i^*(t) = \begin{cases} u_i(t) & \text{if } t \in Z \\ U_i(x_1^*, \ldots, x_I^*) & \text{otherwise} \end{cases}.$$

3 Efficient Tax Rates without TC and under TC

Let us start with the following definition.

Definition 1 *Tax rate $t \in [t_{min}, t_{max}]$ is efficient without TC (under TC, respectively) if there exists no other tax rate $\hat{t} \in [t_{min}, t_{max}]$ such that $u_i(\hat{t}) \geq u_i(t)$ ($u_i^*(\hat{t}) \geq u_i^*(t)$, repectively) for all $i \in \{1, \ldots, I\}$ with strict inequality for some $i \in \{1, \ldots, I\}$.*

It is easy (and by no means novel) to characterise the set of efficient tax rates without TC.

Lemma 2 *The set of efficient tax rates without TC is $[t_{min}, t_1]$.*

The following proposition contains the main theoretical result.

Proposition 1 *If Z contains at least two distinct tax rates, the following hold.*

1) $Z = [z_L, z_U]$ for some z_L and z_U such that $t_{min} \leq z_L < z_U \leq t_1$.

2) Suppose that, in the Nash equilibrium of the game of conflict, $(x_1^*, \ldots, x_I^*)$, the following does not hold: $x_i^* = 0$ for all $i \in \{2, \ldots, I-1\}$, $x_1^* > 0$, $x_I^* > 0$, $U_1(x_1^*, \ldots, x_I^*) = U_1(0, x_2^*, \ldots, x_I^*)$ and $U_I(x_1^*, \ldots, x_I^*) = U_I(x_1^*, \ldots, x_{I-1}^*, 0)$. Then, at least one of the weak inequalities in part 1) is strict.

3) The set of efficient tax rates under TC equals Z.

The possibility that Z consists of a single tax rate seems like an uninteresting knife-edge case. The potentially substantive restriction is that $Z \neq \emptyset$.[11]

The condition in part 2) rules out the case in which only types 1 and I are active in the conflict and each of them is indifferent to being inactive. This also seems like an uninteresting knife-edge case. Moreover, it cannot occur if $\alpha < 1$.[12]

The upshot of Lemma 2 and Proposition 1 is that, under the conditions in the proposition, the set of efficient tax rates under TC is a proper subset of the set of efficient tax rates without TC. Of course, a key question is to what extent these two sets differ. This question will be addressed in the numerical analysis.

4 Calibration

For the numerical analysis, I need to choose the elasticity of labour supply (ϵ), the number of types (I), the wage distribution (the w_i's and p_i's), the preference parameter β, government expenditures (G), and the parameter α in (1).[13]

4.1 Elasticity of Labour Supply

There is considerable controversy in the literature on the appropriate values for the Marshallian and Hicksian elasticities of labour supply with respect to the wage.[14] Table 6 in Keane (2011) reports estimates of these elasticities for males from a wide range of studies. I perform the computations for $\epsilon \in \{0.1, 0.5, 1\}$, which roughly covers the range of estimates of the Hicksian elasticity and the range of positive estimates of the Marshallian elasticity in Keane's Table 6.[15]

[11] If $Z = \emptyset$, all $t \in [t_{\min}, t_{\max}]$ are efficient under TC for the trivial reason that they all get challenged and are, hence, all equally irrelevant.

[12] This follows from Claim 1 in the proof of Lemma 1.

[13] In what follows, all dollar amounts are in 2012 dollars.

[14] Keane (2011) and Saez et al. (2012) provide surveys of this literature.

[15] In Keane's Table 6, the reported estimates of the Hicksian elasticities range between 0.02 and 1.32. Only two of the around two dozen Hicksian-elasticity estimates are above one. The reported

4.2 Number of Types

I set $I = 3$. This can be justified based on the following two considerations which pull in opposite directions: (i) the more types there are, the better one can approximate the empirical distribution of wages, but (ii) a large number of types can lead to some types i and j (where $i \neq j$) being sufficiently similar that $t_i = t_j$. It turns out that, with $I = 4$, one obtains $t_3 = t_4 = t_{\min}$ for any $\epsilon \in \{0.1, 0.5, 1\}$. In contrast, with $I = 3$, it will be the case that, for any $\epsilon \in \{0.1, 0.5, 1\}$, $t_i \neq t_j$ for any two distinct types i and j.

4.3 Wage Distribution

Let $F(\cdot, w_1, w_2, w_3, p_1, p_2)$ denote the cumulative density function (CDF) of wages given the parameters $(w_1, w_2, w_3, p_1, p_2)$. I choose $(w_1, w_2, w_3, p_1, p_2)$ by minimising the L^2 distance between F and an empirical CDF of wages, H.[16] The latter is constructed based on an empirical distribution of hourly wages for individuals between the ages of 25 and 60 in the United States in 2021 obtained from Heathcote et. al (2023). The details of how H is constructed are provided in the appendix.

Table 1 presents the result. The table also shows each type's optimal tax rate, t_i, for each $\epsilon \in \{0.1, 0.5, 1\}$. Note that $t_i \neq t_j$ for any two distinct types i and j.

estimates of the Marshallian elasticities range between -0.47 and 0.7, with an average value of 0.06. For the utility function I use, $\epsilon \leq 0$ does not make sense. (For $\epsilon = 0$ and $\epsilon = -1$, utility is not defined; for ϵ such that $\epsilon < 0$ and $\epsilon \neq -1$, $l_i(t) = \infty$ if $w_i > 0$ and $t < 1$.)

[16] That is, I choose $(w_1, w_2, w_3, p_1, p_2)$ by numerically solving $\min_{\tilde{w}_1, \tilde{w}_2, \tilde{w}_3, \tilde{p}_1, \tilde{p}_2} \int_0^\infty (F(w, \tilde{w}_1, \tilde{w}_2, \tilde{w}_3, \tilde{p}_1, \tilde{p}_2) - H(w))^2 dw$. I verify that this problem has a unique solution by initiating the numerical minimisation algorithm from ten different seeds.

	Type 1	Type 2	Type 3
Hourly wage	\$12.5	\$27.2	\$55.8
Proportion of population	0.41	0.37	0.22
Optimal tax rate given $\epsilon = 0.1$	0.85	0.25	0.16
Optimal tax rate given $\epsilon = 0.5$	0.59	0.21	0.1
Optimal tax rate given $\epsilon = 1$	0.46	0.22	0.09

Table 1: Calibrated distribution of types and types' optimal tax rates.

4.4 The Parameter β

I set the parameter β as follows. I assume that the tax schedule individuals face in reality is of the form $\hat{\tau}(y) = y - e^{1.71666} y^{0.818}$.[17] Then, I set β to equal the solution to $\sum_{i=1}^{I} p_i \left(\frac{0.818 e^{1.71666} w_i^{0.818}}{\beta} \right)^{\frac{\epsilon}{1+\epsilon-0.818\epsilon}} = 1616$. The left-hand side is the average hours worked by the three types in the model if facing the tax schedule $\hat{\tau}(\cdot)$.[18] The right hand-side is the average hours individuals between the ages of 25 and 60 in the United States worked in 2021.[19]

4.5 Government Consumption Per Capita

According to the World Inequality Database, US national income per individual over age 20 in 2021 was \$70,067.[20] According to Piketty, Saez, and Zucman (2018), total (i.e., federal, state, and local) government consumption in the US has been around 18 percent of national income since the end of World War II. I assume that (i) the labour share in national income is sixty percent[21] and (ii) the share of government

[17] This is the tax schedule over income (overall income, not just labour income) estimated by Heathcote et al. (2017) for the United States.

[18] It is straightforward to show that, given $\hat{\tau}(\cdot)$, the optimal labour supply of type i equals $\left(\frac{0.818 e^{1.71666} w_i^{0.818}}{\beta} \right)^{\frac{\epsilon}{1+\epsilon-0.818\epsilon}}$.

[19] To be precise, 1616 is the average of the variable "thours2" for 2021 in the file "cps_sampleC.dta" provided in the replication materials for Heathcote et. al (2023).

[20] The number is \$80,860 in 2021 dollars, which I converted into 2012 dollars using the Bureau of Labor Statistics inflation calculator at https://data.bls.gov/cgi-bin/cpicalc.pl

[21] The Federal Reserve Bank of St. Louis reports on its website that the share of labour compensation in GDP was 0.597 in 2019. See https://fred.stlouisfed.org/series/LABSHPUSA156NRUG

		$\epsilon = 0.1$	$\epsilon = 0.5$	$\epsilon = 1$
$[t_{\min}, t_1]$		$[0.16, 0.85]$	$[0.12, 0.59]$	$[0.09, 0.46]$
	$\alpha = 0.1$	$[0.36, 0.45]$	$[0.24, 0.31]$	$[0.19, 0.25]$
Z	$\alpha = 0.25$	$[0.35, 0.49]$	$[0.23, 0.33]$	$[0.17, 0.25]$
	$\alpha = 0.5$	$[0.32, 0.55]$	$[0.2, 0.35]$	$[0.14, 0.25]$
	$\alpha = 0.1$	0.13	0.15	0.15
$\frac{z_U - z_L}{t_1 - t_{\min}}$	$\alpha = 0.25$	0.19	0.21	0.2
	$\alpha = 0.5$	0.32	0.31	0.3

Table 2: Efficient tax rates without TC ($[t_{\min}, t_1]$) and under TC ($Z = [z_L, z_U]$), as well as the ratio $\frac{z_U - z_L}{t_1 - t_{\min}}$ for different values of α and ϵ.

expenditures financed from taxes on labour income equals the labour share in national income. Thus, I set $G = 70,067 \times 0.18 \times 0.6 \approx \$7,567$.

4.6 Parameter in the Contest Success Function

I am not aware of empirical evidence that could provide guidance for choosing α, especially given that this parameter is likely to be context-specific. Lemma 1 motivates the restriction $\alpha \in (0, 0.5]$. With the aim of covering much of this range, I perform the computations for $\alpha \in \{0.1, 0.25, 0.5\}$.

5 Results

The top part of Table 2 displays the interval $[t_{\min}, t_1]$ for different values of ϵ. The middle part presents, for different combinations of α and ϵ, the set Z, which in all cases turns out to be of the form $Z = [z_L, z_U]$. The bottom part shows, for different combinations of α and ϵ, the ratio $\frac{z_U - z_L}{t_1 - t_{\min}}$, which captures how much smaller the set of efficient tax rates under TC is than the set of efficient tax rates without TC.[22]

[22]To compute Z for given α and ϵ, I needed to compute the Nash equilibrium of the game of conflict. The latter was computed by numerically solving the system of equations (4) given in the proof of Lemma 1.

Table 2 reveals the following.

Finding 1 *For all combinations of* $\alpha \in \{0.1, 0.25, 0.5\}$ *and* $\epsilon \in \{0.1, 0.5, 1\}$*, the following hold.*

1) Z *is an interval* $[z_U, z_L]$*, where* $t_{min} < z_L < z_U < t_1$*.*

2) The length of Z *is in the range 0.06-0.15.*

3) The ratio $\frac{z_U - z_L}{t_1 - t_{min}}$ *is in the range 0.13-0.32.*

The main message of this finding is that TC dramatically shrinks the set of efficient tax rates.

6 Concluding Remarks

My guiding principle in setting up the model was simplicity. I took a simple model of taxation and appended to it a simple, essentially off-the-shelf model of conflict. Apart from providing theoretical and numerical tractability, this guiding principle helps to guard against the danger that I have devised the model to produce a dramatic result.

The flip side is that the model is unrealistic in many respects. Notably, it exhibits static labour supply; identical-across-individuals, quasilinear, constant-elasticity-of-labour-supply preferences over consumption and labour; risk-neutrality for lotteries over consumption; a restriction to affine taxation; a particular technology of conflict (as captured by the U_i functions) that hardly does justice to actual conflict in society; exogenously given groups in the game of conflict that are taken to coincide with wage types (which entailed approximating the empirical distribution of wages with three types); lack of free-riding or miscoordination within groups; no limits on expenditures on conflict; in case of conflict, irrelevance of the initial tax rate and impossibility to challenge the tax rate set by the winner; and a restriction $\alpha \leq 0.5$, which is imposed for convenience (to guarantee a unique Nash equilibrium) rather than based on empirical evidence.

In the end, I view my results as proof of concept that TC can have a substantial impact on the set of efficient tax schedules over labour income. Hopefully, this will encourage other researchers to incorporate TC into their models of taxation.

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Appendix A: Comments on the Absence of Budget Constraints in the Game of Conflict

I allow expenditures in the game of conflict to be arbitrarily large. I do so because (i) it is not clear what upper limits to impose on them[23] and (ii) such limits open up the possibility of corner solutions which would complicate the numerical analysis (especially given that $\alpha \leq 0.5$ may no longer guarantee the existence of a unique Nash equilibrium). Nevertheless, this assumption seems particularly egregious as it ignores the very realistic possibility that high-wage types are likely to have access to more resources that can be devoted to conflict.

Having said that, this assumption by itself probably does not negate the main message of the paper for two reasons. First, Proposition 1 does not rely on the level of the Nash equilibrium expenditures (only on the existence of a unique Nash equilibrium).

Second, it turns out that, for all combinations of α and ϵ considered in the numerical analysis, type 1 challenges tax rates below z_L, type 3 challenges tax rates above z_U, and type 2 challenges tax rates above some threshold that is strictly greater than z_U so that challenges by type 2 do not affect Z. Introducing constraints on expenditures on conflict that are looser for higher types (i) is likely to discourage type 1 from challenging, which would push z_L down, (ii) is likely to encourage type 3 to challenge more, which would push z_U down, and (iii) may cause type 2 to start challenging tax rates that go unchallenged by the other types. Effect (i) makes Z longer while effects

[23] There are three potential avenues for imposing such limits. First, one could assume that each type has an endowment from which to make expenditures on conflict. However, in any quantitative implementation, it would be unclear how to choose these endowments. Second, one could allow each type to borrow up to its worst-case (in terms of who wins the conflict) future post-tax income. Third, one could allow each type to borrow up to a limit that equals the type's expected future post-tax income. In this case the borrowing limit is endogenous–it affects the x_i's which affect types' probabilities of winning which, in turn, affect expected post-tax labour income.

(ii) and (iii) make it shorter, so that the net effect on the length of Z is ambiguous. However, even if effect (i) is maximal (i.e., z_L gets pushed down all the way to $t_{\min}$) while effects (ii) and (iii) are nil, Z would still be much smaller than $[t_{\min}, t_1]$ because, for any combination of α and ϵ, z_U is much smaller than t_1 in Table 2.[24]

Appendix B: The Empirical CDF of Wages

To construct the empirical CDF of wages, H, I proceed as follows. First, I obtain from Heathcote et. al (2023) an empirical distribution of hourly wages for individuals between the ages of 25 and 60 in the United States in 2021.[25,26] Next, I calculate the following percentiles of this distribution: 0 (i.e., lowest wage in the data), 5, 10, ..., 90, 91, 92, ..., 99, 99.1, 99.2, ..., 100 (i.e., highest wage in the data). Then, I obtain a preliminary CDF of wages, $\hat{H}$, as a piecewise function that linearly interpolates between these percentiles.[27] Finally, I obtain H by assuming that ten percent of the population have zero wage and rescaling $\hat{H}$ accordingly, i.e., $H(w) = 0.1 + 0.9\hat{H}(w)$ for $w \in [0, \infty)$. I add zero-wage individuals because the empirical distribution of hourly wages excludes individuals who work zero hours, many of whom presumably

[24] A caveat here is the following. I cannot rule out that effects (i)-(iii) happen to shrink Z down to the empty set. If that happens, all $t \in [t_{\min}, t_{\max}]$ become efficient under TC. See footnote 11.

[25] Wages are computed as $\frac{\text{annual labour income + annual self-employment income}}{\text{annual hours worked}}$, where the denominator is based on the "new hours" measure, which the authors claim is superior. The computations exclude individuals who worked fewer than 260 hours in the year (the vast majority of these having worked zero hours so that wages cannot be computed). For details, see the definitions of labour income and self-employment income as well as the explanations for Figures 6 and 7 in Appendix A of their paper. Their paper itself reports insufficient data for my purposes, so I obtained the distribution of wages from the variable "new_wage" in the "cps_sampleC.dta" file provided in their replication materials.

[26] Some summary statistics of this empirical distribution are as follows: mean = 34.5; standard deviation = 40.7; (lowest wage, 50-th percentile, 90-th percentile, 99-th percentile, 99.9th percentile, highest wage) = (3.6, 25, 60.6, 173.1, 528.9, 1375).

[27] For example, if wages w' and w'' are the 5-th and 10-th percentiles, respectively, then, $\hat{H}(w) = 5 + \frac{10-5}{w''-w'}(w - w')$ for $w \in [w', w'']$.

would not work whatever the tax schedule.[28]

Appendix B: Proofs

6.1 Proof of Lemma 1

For future reference, note that at any (x_i, x_{-i}) such that $x_i > 0$,[29] we have

$$\frac{\partial U_i}{\partial x_i}(x_i, x_{-i}) = \frac{\alpha p_i^\alpha}{x_i^{1-\alpha}\left(\sum_{j=1}^{I} p_j^\alpha x_j^\alpha\right)^2} \sum_{j \neq i} p_j^\alpha x_j^\alpha (u_i(t_i) - u_i(t_j)) - 1. \tag{2}$$

Note also that the assumption $t_i \neq t_j$ for all i and j such that $i \neq j$ implies $u_i(t_i) > u_i(t_j)$ for all i and j such that $i \neq j$. The proof below implicitly relies on the latter set of inequalities.

The proof proceeds via a sequence of claims.

Claim 1 *Assume $\alpha < 1$. Given any $i \in \{1, \ldots, I\}$ and x_{-i}, $0 \notin \arg\max_{x_i \geq 0} U_i(x_i, x_{-i})$.*

Proof:

If $x_{-i} = (0, \ldots, 0)$, $x_i = 0$ cannot be a best response for type i because that type can increase its probability of winning in a discrete fashion (namely, from 1/I to 1) by making an infinitesimal strictly positive expenditure on conflict.

If $x_{-i} \neq (0, \ldots, 0)$, $U_i(x_i, x_{-i})$ is continuous in x_i at $x_i = 0$. But then

$$\lim_{x_i \downarrow 0} \frac{\partial U_i}{\partial x_i}(x_i, x_{-i}) = \frac{\alpha p_i^\alpha}{x_i^{1-\alpha}\left(\sum_{j=1}^{I} p_j^\alpha x_j^\alpha\right)^2} \sum_{j \neq i} p_j^\alpha x_j^\alpha (u_i(t_i) - u_i(t_j)) - 1 = \infty \tag{3}$$

implies that $x_i = 0$ cannot be a best response to x_{-i}. Q.E.D.

[28] Nineteen percent of individuals in the data for 2021 in Heathcote et. al (2023) work zero hours. These individuals do not work given the actual tax schedule they are facing. I am effectively assuming that around half of them would not work given any tax schedule, and I am ignoring the other half.

[29] As usual, $x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_I)$. Also, with the customary abuse of notation, (x_i, x_{-i}) is to be read as $(x_1, \ldots, x_I)$.

Claim 2 *Assume $\alpha < 1$. $(x_1^*, \ldots, x_I^*)$ is a Nash equilibrium if and only if it solves*

$$\frac{\partial U_i}{\partial x_i}(x_i, x_{-i}) = 0, \forall i \in \{1, \ldots, I\}. \tag{4}$$

Proof:

Suppose $(x_1^*, \ldots, x_I^*)$ is a Nash equilibrium. By Claim 1, $x_i^* > 0$. Therefore, $(x_1^*, \ldots, x_I^*)$ must satisfy the first-order conditions (4).

In the other direction, assume that $(x_1^*, \ldots, x_I^*)$ satisfies the first-order conditions (4). Fix $i \in \{1, \ldots, I\}$. Note that we must have $x_i^* > 0$. Given that $\frac{\partial U_i}{\partial x_i}(\cdot, x_{-i}^*)$ is strictly decreasing on $(0, \infty)$, $\frac{\partial U_i}{\partial x_i}(x_i^*, x_{-i}^*) = 0$ is sufficient to guarantee that $U_i(x_i^*, x_{-i}^*) \geq U_i(x_i, x_{-i}^*)$ for all $x_i > 0$. Thus either $U_i(x_i^*, x_{-i}^*) \geq U_i(x_i, x_{-i}^*)$ for all $x_i \geq 0$ or $U_i(0, x_{-i}^*) \geq U_i(x_i, x_{-i}^*)$ for all $x_i \geq 0$. The latter possibility is ruled out by Claim 1. Q.E.D.

Claim 3 *If $\alpha < 1$, the system of equations (4) has a solution.*

Proof:

Let $\Delta = \{s \in \mathbb{R}^I | \sum_{i=1}^I s_i = 1 \text{ and } s_i \geq 0, \forall i\}$. Given $s \in \Delta$, define

$$R(s) = \left(\sum_{i=1}^I \left(\alpha p_i \sum_{j \neq i} s_j (u_i(t_i) - u_i(t_j)) \right)^{\frac{\alpha}{1-\alpha}} \right)^{1-\alpha}. \tag{5}$$

Observing that $R(s) > 0$, define the function $\phi : \Delta \to \Delta$ by

$$\phi_i(s) = \frac{1}{R(s)^{\frac{1}{1-\alpha}}} \left(\alpha p_i \sum_{j \neq i} s_j (u_i(t_i) - u_i(t_j)) \right)^{\frac{\alpha}{1-\alpha}}, \forall i \in \{1, \ldots, I\}.$$

Given that Δ is compact and ϕ is continuous, there exists s^* such that $\phi(s^*) = s^*$ (by Brauwer's fixed point theorem).

Now, let $(x_1^*, \ldots, x_I^*)$ be defined by

$$x_i^* = \left(\frac{s_i^* R(s^*)}{p_i^\alpha} \right)^{\frac{1}{\alpha}}. \tag{6}$$

Using (2), the first-order conditions (4) evaluated at $(x_1^*, \ldots, x_I^*)$ can be written as[30]

$$\frac{1}{R(s^*)^{\frac{1}{1-\alpha}}} \left(\alpha p_i \sum_{j \neq i} s_j^* (u_i(t_i) - u_i(t_j)) \right)^{\frac{\alpha}{1-\alpha}} = s_i^*, \forall i \in \{1, \ldots, I\}. \tag{7}$$

But the left-hand side is $\phi_i(s^*)$ which equals s_i^* given that s^* is a fixed point of ϕ. Thus, $(x_1^*, \ldots, x_I^*)$ solves the system of equations (4). Q.E.D.

Claim 4 *If $\alpha \leq 1/2$, the system of equations (4) has a unique solution.*

Proof:

Define $R(x_1, \ldots, x_I) = \sum_{j=1}^{I} p_j^\alpha x_j^\alpha$ and $\sigma_i(x_1, \ldots, x_I) = \frac{p_i^\alpha x_i^\alpha}{R(x_1, \ldots, x_I)}$.

Assume there are two solutions to the first-order conditions (4), $(\bar{x}_1, \ldots, \bar{x}_I)$ and $(\hat{x}_1, \ldots, \hat{x}_I)$ such that $(\bar{x}_1, \ldots, \bar{x}_I) \neq (\hat{x}_1, \ldots, \hat{x}_I)$.[31] Without loss of generality, assume $R(\bar{x}_1, \ldots, \bar{x}_I) \geq R(\hat{x}_1, \ldots, \hat{x}_I)$.

Note that the first-order conditions (4) evaluated at $(\bar{x}_1, \ldots, \bar{x}_I)$ and $(\hat{x}_1, \ldots, \hat{x}_I)$ can be written, respectively, as

$$\alpha \sigma_i(\bar{x}_1, \ldots, \bar{x}_I) \sum_{j \neq i} \sigma_j(\bar{x}_1, \ldots, \bar{x}_I)(u_i(t_i) - u_i(t_j)) = \bar{x}_i, \forall i \in \{1, \ldots, I\} \tag{8}$$

$$\alpha \sigma_i(\hat{x}_1, \ldots, \hat{x}_I) \sum_{j \neq i} \sigma_j(\hat{x}_1, \ldots, \hat{x}_I)(u_i(t_i) - u_i(t_j)) = \hat{x}_i, \forall i \in \{1, \ldots, I\}. \tag{9}$$

[30] The equations below are equivalent to the first-order conditions (4) at $(x_1^*, \ldots, x_I^*)$ if $x_i^* > 0, \forall i$. The latter inequality holds for the following reason. $s_i = 0$ implies $s_j > 0$ for some $j \neq i$ which, in turn, implies $\phi_i(s) > 0$. Thus, $s_i = 0$ cannot hold at a fixed point of ϕ. Thus, $s_i^* > 0$ which, by the definition of x_i^* in (6), implies $x_i^* > 0$.

[31] By Claim 1, each type's expenditure on conflict is strictly positive in any Nash equilibrium. Hence, $\sigma_i(\bar{x}_1, \ldots, \bar{x}_I)$ and $\sigma_i(\hat{x}_1, \ldots, \hat{x}_I)$ are well-defined and strictly positive.

Based on conditions (8) and (9), $\sigma_i(\bar{x}_1, \dots, \bar{x}_I) = \sigma_i(\hat{x}_1, \dots, \hat{x}_I)$ for all i implies $(\bar{x}_1, \dots, \bar{x}_I) = (\hat{x}_1, \dots, \hat{x}_I)$, a contradiction. Hence, $\sigma_i(\bar{x}_1, \dots, \bar{x}_I) \neq \sigma_i(\hat{x}_1, \dots, \hat{x}_I)$ for some i.

Let $k \in \{1, \dots, I\}$ be such that $\frac{\sigma_k(\bar{x}_1,\dots,\bar{x}_I)}{\sigma_k(\hat{x}_1,\dots,\hat{x}_I)} \geq \frac{\sigma_j(\bar{x}_1,\dots,\bar{x}_I)}{\sigma_j(\hat{x}_1,\dots,\hat{x}_I)}$ for any $j \in \{1, \dots, I\}$. Given that $\sum_{j=1}^{I} \sigma_j(\bar{x}_1, \dots, \bar{x}_I) = \sum_{j=1}^{I} \sigma_j(\hat{x}_1, \dots, \hat{x}_I) = 1$, $\sigma_k(\bar{x}_1, \dots, \bar{x}_I) > \sigma_k(\hat{x}_1, \dots, \hat{x}_I)$ must hold. Given the latter inequality and $R(\bar{x}_1, \dots, \bar{x}_I) \geq R(\hat{x}_1, \dots, \hat{x}_I)$, it follows that $\frac{\bar{x}_k}{\hat{x}_k} > 1$.

From the first-order conditions (8) and (9), we obtain

$$\begin{aligned}
&\frac{\bar{x}_k}{\hat{x}_k} = \frac{\alpha\sigma_k(\bar{x}_1, \dots, \bar{x}_I)}{\alpha\sigma_k(\hat{x}_1, \dots, \hat{x}_I)} \frac{\sum_{j\neq k} \sigma_j(\bar{x}_1, \dots, \bar{x}_I)(u_i(t_i) - u_i(t_j))}{\sum_{j\neq k} \sigma_j(\hat{x}_1, \dots, \hat{x}_I)(u_i(t_i) - u_i(t_j))} = \\
&\frac{R(\hat{x}_1, \dots, \hat{x}_I)}{R(\bar{x}_1, \dots, \bar{x}_I)} \frac{\bar{x}_k^\alpha}{\hat{x}_k^\alpha} \frac{\sum_{j\neq k} \frac{\sigma_j(\bar{x}_1,\dots,\bar{x}_I)}{\sigma_j(\hat{x}_1,\dots,\hat{x}_I)} \sigma_j(\hat{x}_1, \dots, \hat{x}_I)(u_i(t_i) - u_i(t_j))}{\sum_{j\neq k} \sigma_j(\hat{x}_1, \dots, \hat{x}_I)(u_i(t_i) - u_i(t_j))} < \\
&\frac{R(\hat{x}_1, \dots, \hat{x}_I)}{R(\bar{x}_1, \dots, \bar{x}_I)} \frac{\bar{x}_k^\alpha}{\hat{x}_k^\alpha} \frac{\sum_{j\neq k} \frac{\sigma_k(\bar{x}_1,\dots,\bar{x}_I)}{\sigma_k(\hat{x}_1,\dots,\hat{x}_I)} \sigma_j(\hat{x}_1, \dots, \hat{x}_I)(u_i(t_i) - u_i(t_j))}{\sum_{j\neq k} \sigma_j(\hat{x}_1, \dots, \hat{x}_I)(u_i(t_i) - u_i(t_j))} = \\
&\frac{R(\hat{x}_1, \dots, \hat{x}_I)}{R(\bar{x}_1, \dots, \bar{x}_I)} \frac{\bar{x}_k^\alpha}{\hat{x}_k^\alpha} \frac{\sigma_k(\bar{x}_1, \dots, \bar{x}_I)}{\sigma_k(\hat{x}_1, \dots, \hat{x}_I)} = \\
&\frac{R(\hat{x}_1, \dots, \hat{x}_I)^2}{R(\bar{x}_1, \dots, \bar{x}_I)^2} \frac{\bar{x}_k^{2\alpha}}{\hat{x}_k^{2\alpha}} \leq \left(\frac{\bar{x}_k}{\hat{x}_k}\right)^{2\alpha}.
\end{aligned}$$

Putting together $\frac{\bar{x}_k}{\hat{x}_k} > 1$ and $\frac{\bar{x}_k}{\hat{x}_k} < \left(\frac{\bar{x}_k}{\hat{x}_k}\right)^{2\alpha}$, it follows that $2\alpha > 1$, i.e. $\alpha > 1/2$. Q.E.D.

6.2 Proof of Lemma 2

Note that (i) for each type i, $u_i(\cdot)$ is single-peaked with peak at t_i (i.e., strictly increasing to the left of t_i and strictly decreasing to the right of t_i) and (ii) $t_{\min} = t_I \leq t_{I-1} \leq \ldots \leq t_1$.[32] As a result, (a) any tax rate above t_1 is strictly worse than t_1

[32]Point (i) can be established by examining the derivative of $u_i(\cdot)$. Point (ii) follows from the expression for t_i given in footnote 6.

for all types and (b) for any $t \in [t_{\min}, t_1]$, any move to a higher tax rate makes type t_I worse off and any move to a lower tax rate makes type t_1 strictly worse off. Q.E.D.

6.3 Proof of Proposition 1

In the proof, I will repeatedly and without explicit mention make use of the statements in the first sentence in the proof of Lemma 2. By way of notation, let $(x_1^*, \ldots, x_I^*)$ denote the Nash equilibrium of the game of conflict.

Proof of part 1):

Take tax rates t' and t'' such that $t' < t''$ and $t', t'' \in Z$, and take $t \in (t', t'')$. For any type i, $u_i(t) \geq \min(u_i(t'), u_i(t''))$. Hence, no type challenges t so that $t \in Z$. Thus, Z is a nonempty interval.

Next, let $Z_i = \{t \in [t_{\min}, t_{\max}] | u_i(t) \geq U_i(x_1^*, \ldots, x_I^*)\}$. That is, Z_i denotes the set of feasible tax rates that do not get challenged by type i. Given the continuity of $u_i(\cdot)$, Z_i is closed. Given that $Z = \bigcap_{i=1}^{I} Z_i$, Z is closed.

Finally, type I is strictly worse off under any tax rate $t > t_1$ than if it challenges, sets $x_I = 0$, and the worst possible type from I's perspective (which is type 1) wins. Hence, $t \notin Z$. Thus, $Z \subseteq [t_{\min}, t_1]$.[33] Q.E.D.

Proof of part 2):

Consider the following exhaustive cases.

Case 1: $x_i^* > 0$ for some $i \in \{2, \ldots, I-1\}$.

[33] $Z \subseteq [t_{\min}, t_1]$ implies that $t_{\min} < t_1$, which will be used without explicit mention in the rest of the proof.

In this case, we have

$$U_1(x_1^*, \ldots, x_I^*) \geq U_1(0, x_2^* \ldots, x_I^*) > \min_{j \in \{1,\ldots,I\}} u_1(t_j) = u_1(t_I) = u_1(t_{\min})$$

$$U_I(x_1^*, \ldots, x_I^*) \geq U_I(x_1^*, \ldots, x_{I-1}^*, 0) > \min_{j \in \{1,\ldots,I\}} u_I(t_j) = u_I(t_1),$$

where the strict inequality in the first (respectively, second) line follows from the fact that, with expenditures on conflict $(0, x_2^* \ldots, x_I^*)$ (respectively, $(x_1^*, \ldots, x_{I-1}^*, 0)$), there is a positive probability that type $i \in \{2, \ldots, I-1\}$ wins, which is not the worst possible outcome for type 1 (respectively, I).

By the continuity of $u_1(\cdot)$, type 1 rejects all tax rates close enough to $t_{\min}$. Similarly, by the continuity of $u_I(\cdot)$, type I rejects all tax rates close enough to t_1. Thus, tax rates close enough to $t_{\min}$ and t_1 are not in Z.

Case 2: $x_i^* = 0$ for all $i \in \{2, \ldots, I-1\}$ and either $x_1^* = 0$ or $x_I^* = 0$.

If $x_1^* = 0$, type I does not have a best-response. In particular, $x_I = 0$ cannot be a best-response because an infinitesimal increase in expenditure on conflict increases type I's probability of winning in a discrete fashion, and $x_I > 0$ cannot be a best-response because making a smaller (but still positive) expenditure on conflict does not reduce type I's probability of winning. If $x_I^* = 0$, type 1 does not have a best-response for similar reasons. Thus, this case is inconsistent.

Case 3: $x_i^* = 0$ for all $i \in \{2, \ldots, I-1\}$, $x_1^* > 0$, $x_I^* > 0$, $U_1(x_1^*, \ldots, x_I^*) > U_1(0, x_2^*, \ldots, x_I^*)$.

In this case, we have

$$U_1(x_1^*, \ldots, x_I^*) > U_1(0, x_2^* \ldots, x_I^*) \geq \min_{j \in \{1,\ldots,I\}} u_1(t_j) = u_1(t_I) = u_1(t_{\min}).$$

By the continuity of $u_1(\cdot)$, type 1 rejects all tax rates close enough to $t_{\min}$. Thus, tax rates close enough to $t_{\min}$ are not in Z.

Case 4: $x_i^* = 0$ for all $i \in \{2, \ldots, I-1\}$, $x_1^* > 0$, $x_I^* > 0$, $U_I(x_1^*, \ldots, x_I^*) > U_I(x_1^*, \ldots, x_{I-1}^*, 0)$.

In this case, we have

$$U_I(x_1^*, \ldots, x_I^*) > U_I(x_1^*, \ldots, x_{I-1}^*, 0) \geq \min_{j \in \{1,\ldots,I\}} u_I(t_j) = u_I(t_1).$$

By the continuity of $u_I(\cdot)$, type I rejects all tax rates close enough to t_1. Thus, tax rates close enough to t_1 are not in Z. Q.E.D.

Proof of part 3):

Let z_L and z_U denote the lower and upper end point, respectively, of the interval Z. Take any tax rate $\tilde{t} \in [t_{\min}, t_{\max}]$ such that $\tilde{t} \notin [z_L, z_U]$. Tax rate $\tilde{t}$ gets challenged so that each type i obtains expected utility $U_i(x_1^*, \ldots, x_I^*)$. Note that $U_i(x_1^*, \ldots, x_I^*) \leq u_i(t)$ for any $t \in [z_L, z_U]$. Let $A_i = \{t \in [z_L, z_U] | U_i(x_1^*, \ldots, x_I^*) = u_i(t)\}$. Given the shape of $u_i(\cdot)$, A_i consists of at most two points. Hence, $\bigcup_{i=1}^I A_i$ consists of at most $2I$ points. Thus, it is possible to pick $\hat{t} \in [z_L, z_U]$ such that $U_i(x_1^*, \ldots, x_I^*) < u_i(\hat{t}), \forall i \in \{1, \ldots, I\}$. Thus, $\tilde{t}$ is inefficient under TC.

Next pick $t \in [z_L, z_U]$. At any tax rate outside of $[z_L, z_U]$, all types are weakly worse off. Type I is strictly worse off at any tax rate in $[z_L, z_U]$ above t. Type 1 is strictly worse off at any tax rate in $[z_L, z_U]$ below t. Thus, t is efficient under TC. Q.E.D.

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