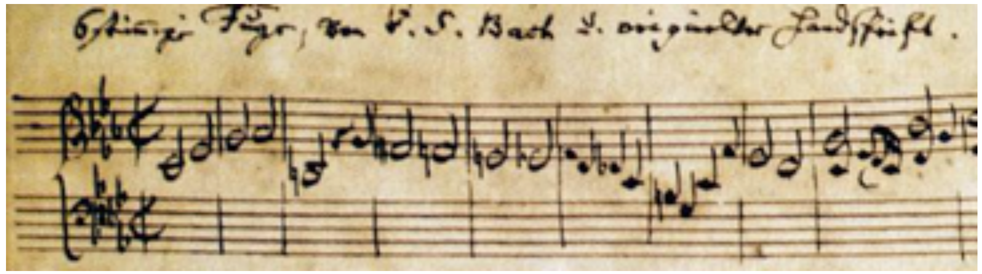
Harmony of scattering amplitudes



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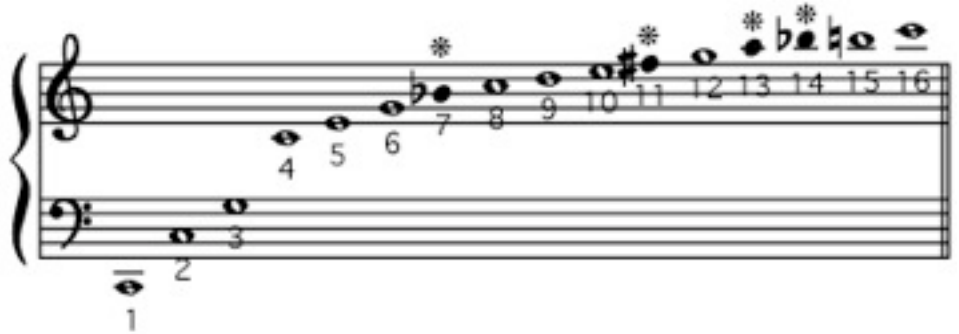
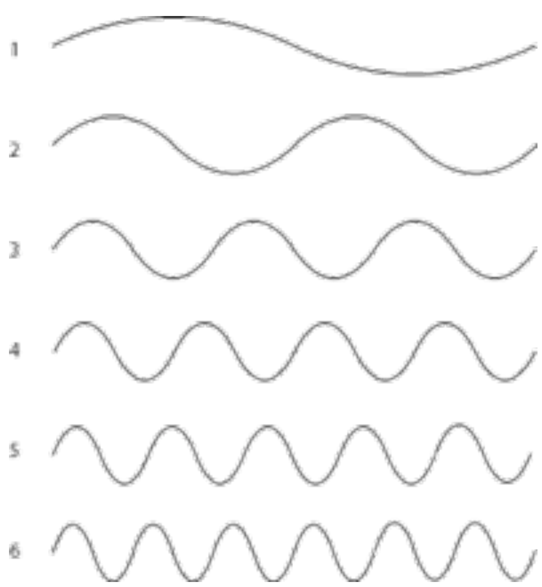
Launch of the School of Physics & Astronomy, 31st January 2012

- In music, certain chords are more pleasant to the ear than others...they are “harmonious”
  - octave, fifth, major third
  - consonant chords
- Can we understand this ?

• Overtones, and the harmonic series





- The 2^{nd} harmonic has twice the frequency of the fundamental

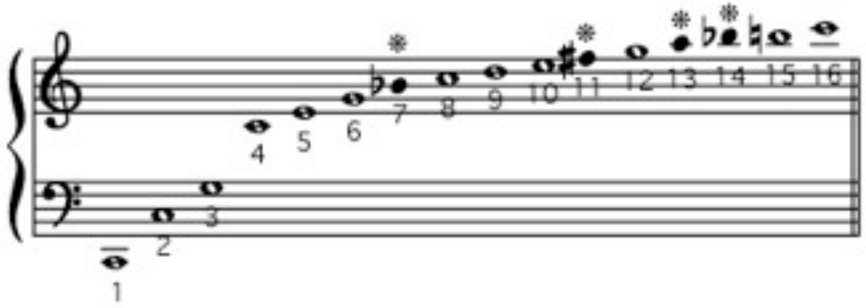
- The 3^{rd} harmonic has 3 times the frequency of the fundamental (and hence 3/2 of the frequency of the 2^{nd} : the fifth)

- The 4^{th} harmonic has 4 times the frequency of the fundamental (and hence 4/3 of the frequency of the 3^{rd} : the fourth)

Regularities in the ratios of the frequencies

Musical hidden structures

- Our first hidden structure...



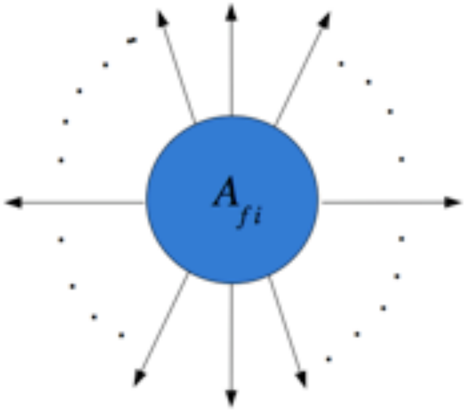
  - overlapping the first notes in the harmonic series gives rise to consonant chords...that is harmony!

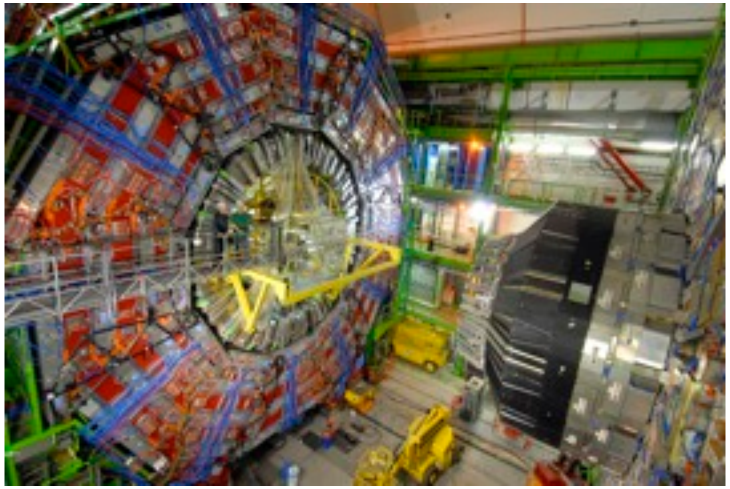
perfect major chord

  - harmony is about hidden structures, or hidden regularities...

Harmony in particle physics

- Scattering amplitudes of elementary particles (e.g. gluons, or gravitons)





  - mathematical quantities related to the probability of the occurrence of a certain scattering process between particles
  - describe processes (and probe physics) at the shortest distance scales possible in a laboratory
  - measured at particle colliders such as the LHC!

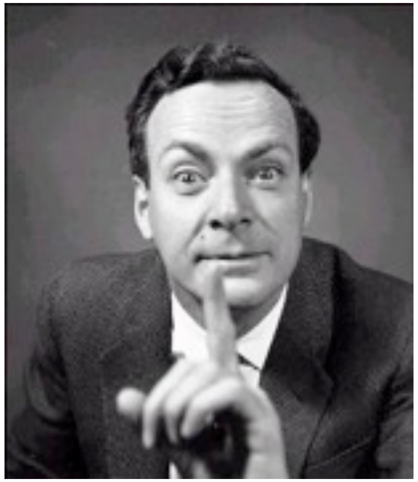
What's special about amplitudes?

- Their simplicity !
  - calculation with textbook methods cumbersome, however final results are often strikingly simple
- Theorem:
  - if you have a simple result, there should be a simple way to get there!

Textbook approach to amplitudes:

Calculate Feynman diagrams !

Textbook approach to amplitudes:



So, what's wrong with them ?

Unwanted complexity

Number of Feynman diagrams for $gg \longrightarrow n\, g$ scattering: (tree level)

☞

n	2	3	4	5	6	7	8
# of diagrams	4	25	220	2485	34300	559405	10525900

Gluon scattering

☞ Result is: $\mathcal{A}(1^{\pm}, 2^{+}, \ldots, n^{+}) =$

FullSimplify[

+ many more pages like this...]

$= 0$!!

Why so simple? Why zero?

The simplest amplitude

(Parke & Taylor, 1986; Berends, Giele 1987; Mangano, Parke, Xu 1988)

$$\mathcal{A}_{\mathrm{MHV}}(1^+,\ldots,i^-,\ldots j^-,\ldots,n^+) = \frac{\langle i\,j\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\,\cdots\langle n\,1\rangle}$$

gluon helicities are a permutation of $--++\ldots+$

Maximally Helicity Violating, or MHV

Back-of-envelope formula...

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where s is the corresponding pole and z is the momentum fraction. The result for particles 2 and 3 nearly parallel, Eq. (5), is only simple because $\mathcal{M}_{n-1}(-+++\cdots)$ is zero to this order in g so that there is no interference term and therefore azimuthal averaging is not required.

The surprise about this result is that all denominators are simple dot products of two external momenta. The Feynman diagrams for n-gluon $(n>5)$ scattering contain propagators $(p_i+p_j+p_k)^2$, $(p_i+p_j+p_k+p_m)^2$, ... These propagators must cancel for Eq. (3) to be correct; this occurs for $n=6$. Of course, Altarelli and Parisi have taught us that many cancellations are expected.

We do not expect such a simple expression for the other helicity amplitudes. Also, we challenge the string theorists to prove more rigorously that Eq. (3) is correct.

Fermilab is operated by the Universities Research Association Inc. under contract with the United States Department of Energy.

[1] E. Eichten, I. Hinchliffe, K. Lane, and C. Quigg, Rev. Mod. Phys. **56**, 579 (1984).
[2] M. T. Grisaru, H. N. Pendleton, and P. van Nieuwenhuizen, Phys. Rev. D **15**, 997 (1977); M. T. Grisaru and H. N. Pendleton, Nucl. Phys. **B124**, 81 (1977).
[3] S. J. Parke and T. R. Taylor, Phys. Lett. **157B**, 81 (1985).
[4] T. Gottschalk and D. Sivers, Phys. Rev. D **21**, 102 (1980); F. A. Berends, R. Kleiss, P. de Causmacker, R. Gastmans, and T. T. Wu, Phys. Lett. **103B**, 124 (1981).
[5] S. J. Parke and T. R. Taylor, Fermilab Report No. Pub-85/118-T, 1985 (to be published); Z. Kunszt, CERN Report No. TH-4319, 1985 (to be published).
[6] Another numerical fact worth mentioning is that to leading order in g but to all orders in N, the amplitude $|\mathcal{M}_{n=6}(--++++)|^2$ is permutation symmetric apart from the factor $(p_1\cdot p_2)^4$. This allows all permutations of this amplitude to be trivially calculated from one such permutation.
[7] G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977).

S.J. Parke & T. R. Taylor , Phys. Rev. Lett. 56

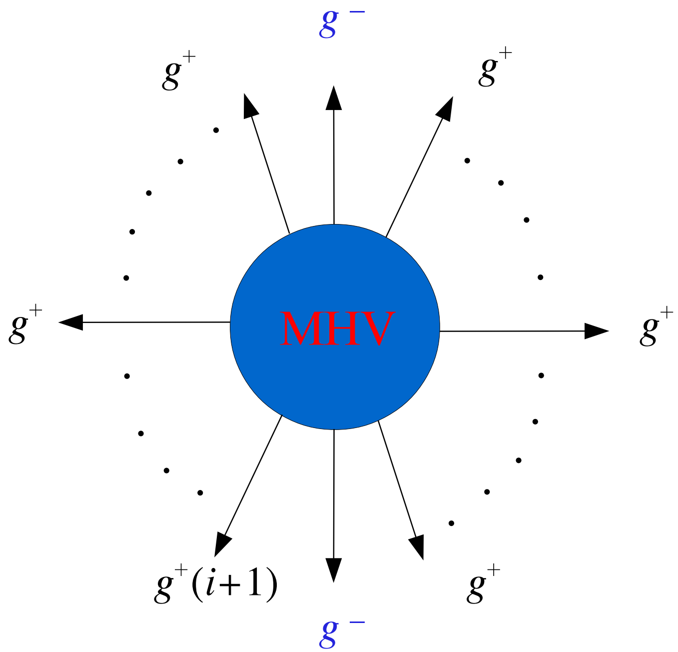
- There are hidden structures in scattering amplitudes (and other physical observables)...

- ...which are not captured by Feynman diagrams

- Need new framework to describe and calculate them

Harmony of the MHV amplitude

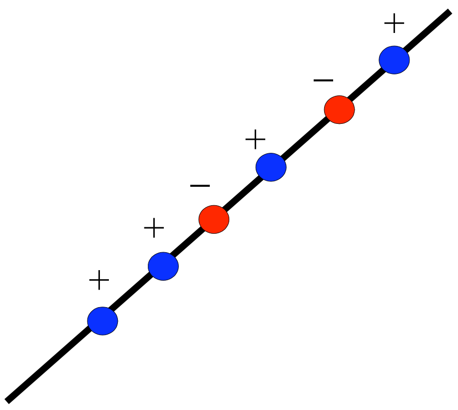
(Witten '03)



transform to

$\longrightarrow$

"twistor space"



- Harmony unveiled in Penrose's twistor space
  - MHV amplitude $\rightarrow$ line in twistor space
  - simplicity in terms of geometry
    - simplest scattering amplitude corresponds to simplest curve

- Geometry in twistor space suggests to use the MHV amplitude as a new “local” vertex
  - New perturbative expansion of gauge theory
  - Twistor space crucial to see the structure
    - lines in twistor space correspond to points in spacetime (Penrose’s incidence relation)

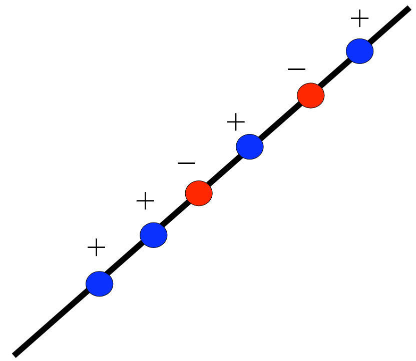
Twistor space

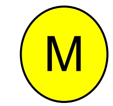
Spacetime

Amplitude | Twistor space structure | MHV diagrams

MHV

−−++....+

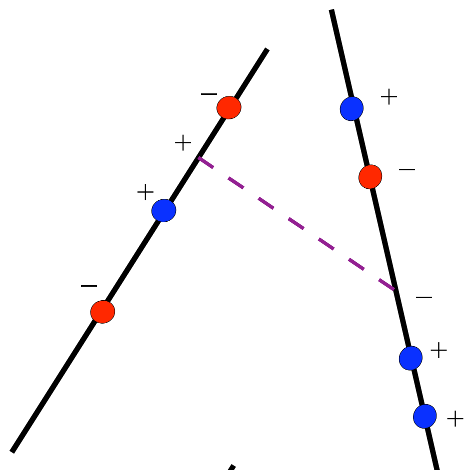




NMHV

(Next-to-MHV)

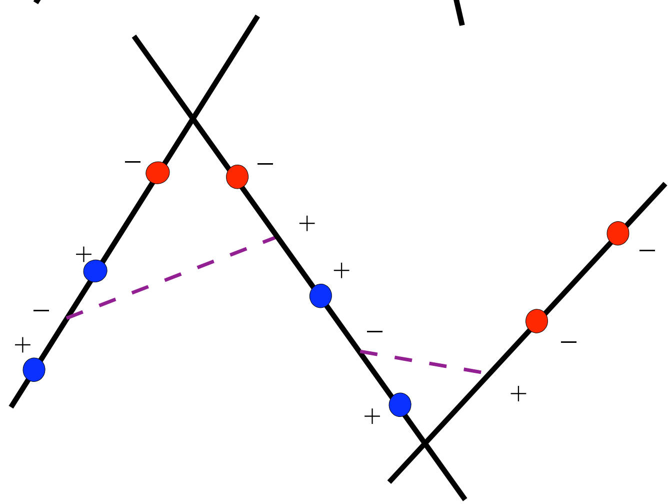
−−−++....+

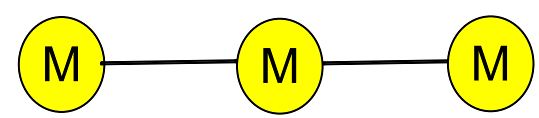


NNMHV

(Next-to-Next-to-MHV)

−−−−++....+

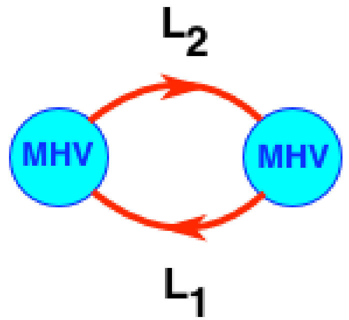




- All amplitudes can be derived using a novel powerful method: MHV diagrams

  - tree level (Cachazo, Svrcek, Witten '04)
  - works beautifully at the quantum level (Brandhuber, Spence, GT, '04)

One-loop MHV amplitude:



Quantum:
particle creation and annihilation

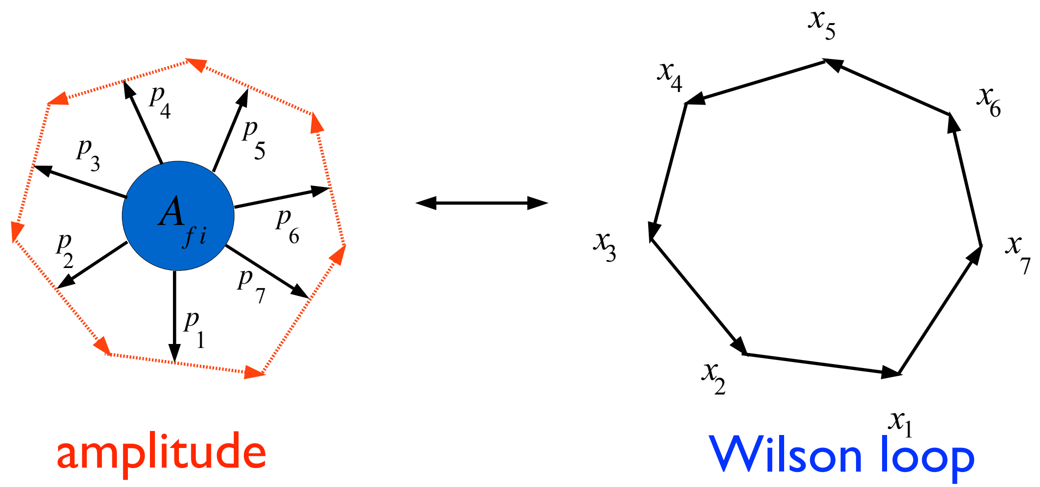
- first novel and phenomenologically relevant result derived with MHV diagrams (Bedford, Brandhuber, Spence, GT, '04)

- ...and this was only the beginning...

- ...since then, a wealth of fascinating new results obtained by many groups worldwide
    - IAS Princeton, Perimeter Institute, UCLA, Brown, Stanford, SLAC, MIT, Michigan, Saclay, Annecy, IHES (Paris), Uppsala, Niels Bohr Institute (Copenhagen), Oxford, Durham, Humboldt (Berlin), Max-Planck-Institut (Potsdam), DESY (Hamburg), ETH Zurich, LNF (Frascati), Turin, Weizmann Institute...
    - more than 30 workshops so far, regular "Amplitude" series (2 at QM, 2 co-organised by QM)

- Some results with key QM involvement:
    - Recursion relations in General Relativity
      (Bedford, Brandhuber, Spence, GT '05)
    - Generalised generalised unitarity for QCD amplitudes
      (Brandhuber, McNamara, Spence, GT '05)
    - Amplitude/Wilson loop duality
      (Alday & Maldacena; Drummond, Korchemsky, Sokatchev; Brandhuber, Heslop GT '07)

Amplitude / Wilson loop duality



- Dualities bring novel conceptual insights...
- Much easier to compute!
    - Wilson loop: one hour on my laptop. Amplitude: one week on the UCLA computer cluster! (Anastasiou, Brandhuber, Heslop, Khoze, Spence, GT ’08)
- Novel, hidden symmetries suddenly become manifest

Summary

- Amplitudes are at the crossroad of exciting subjects in physics and mathematics...
  - string theory & supersymmetric gauge theory
  - pure mathematics, number theory
  - integrability
  - twistor theory
  - phenomenology of particle physics
- Great amount of information exchanged between experts in these areas
  - possibly the key to its success

- Hopefully QM will stay at the forefront of new exciting developments for long!